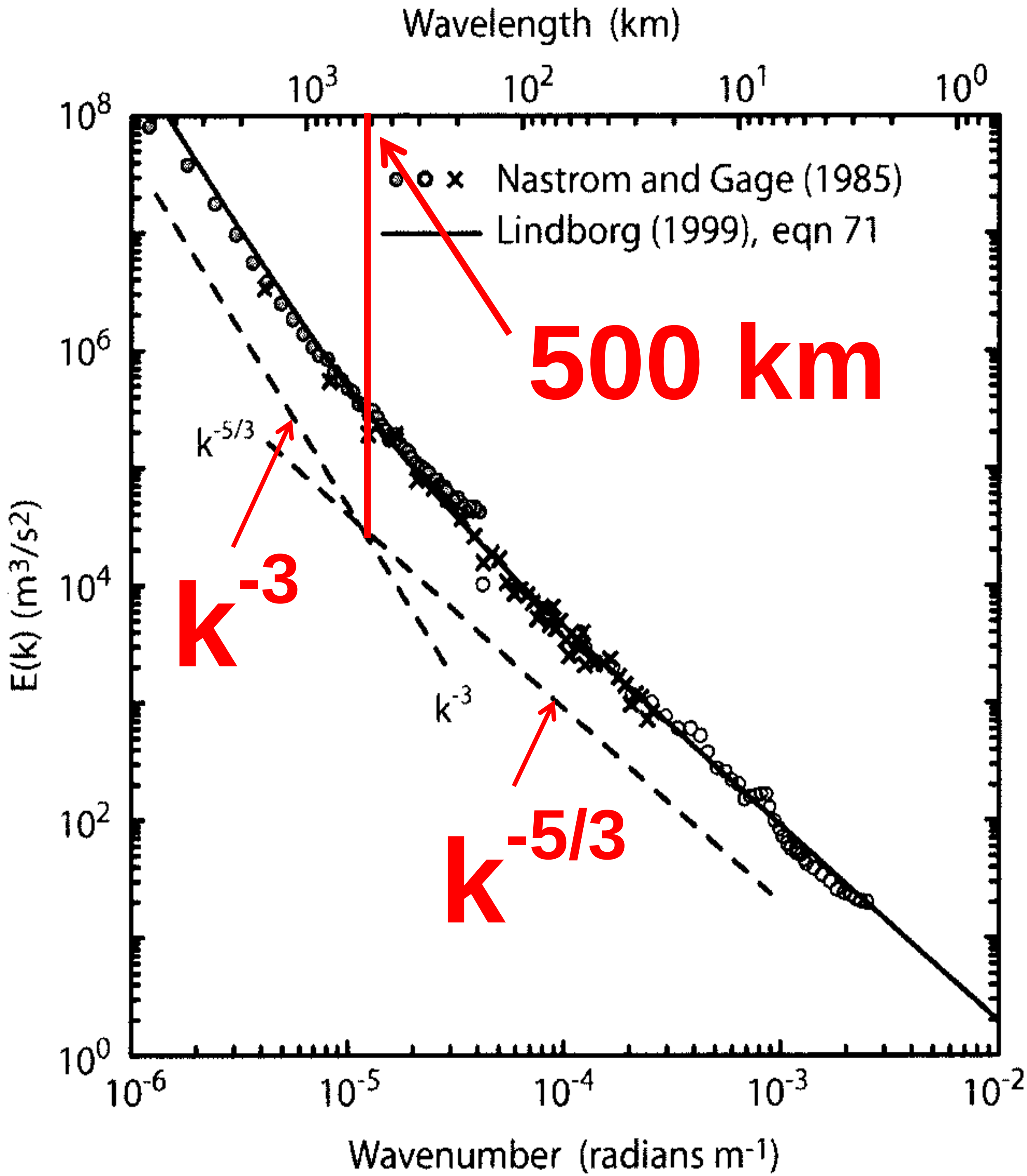
Betydelsen av sammanväxten av data-assimilation och ensemble-teknik för Nowcasting

Åke Johansson
SMHI

Prediktabilitet



Wavelength (km)
10^3
10^2
10^1
10^0
10^8
10^6
10^4
10^2
10^0
Nastrom and Gage (1985)
Lindborg (1999), eqn 71
500 km
$k^{-5/3}$
k^{-3}
k^{-3}
$k^{-5/3}$
E(k) (m^3/s^2)
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
Wavenumber (radians m^{-1})

Allerstädes närvarande

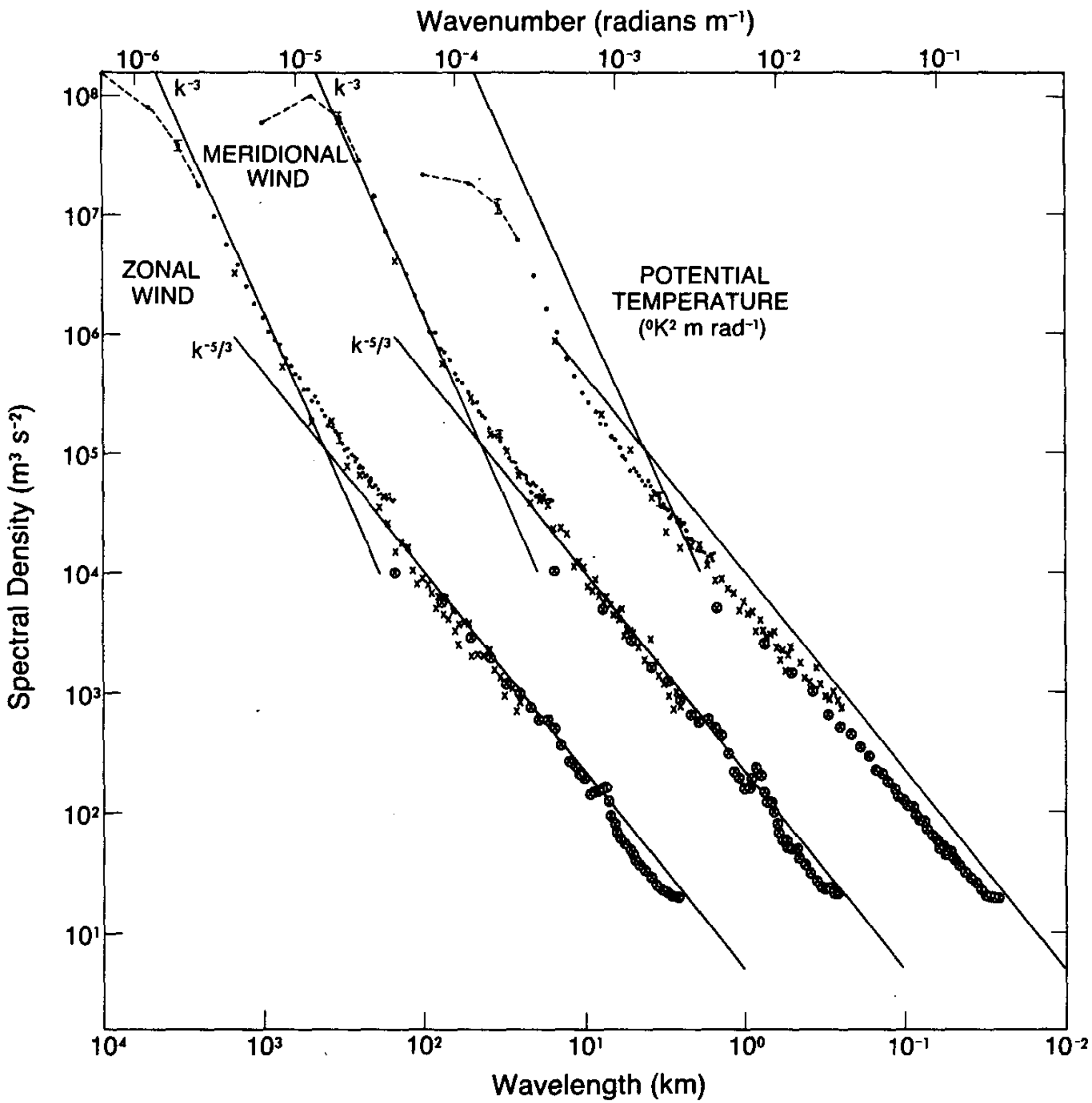
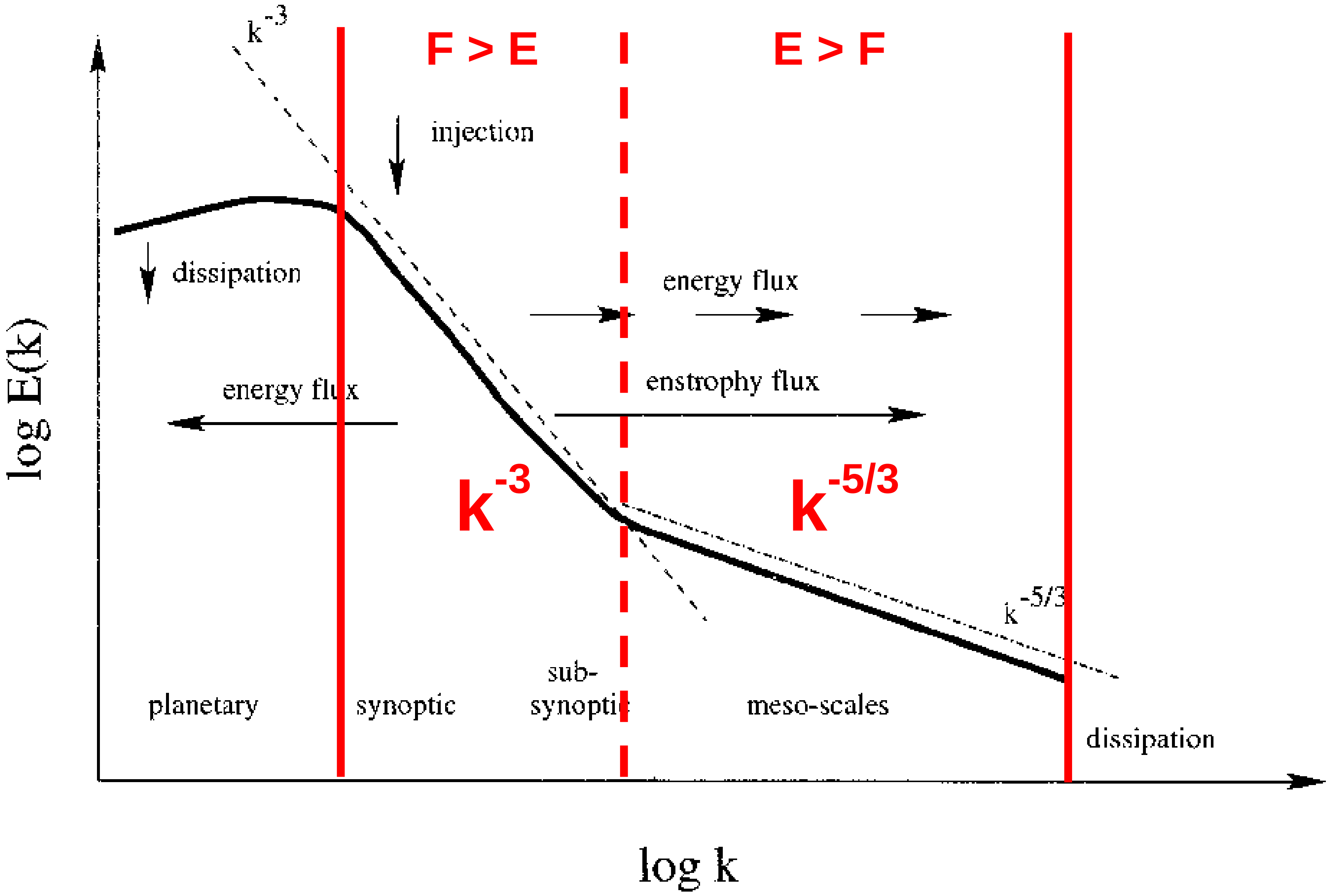


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes −3 and −5/3 are entered at the same relative coordinates for each variable for comparison.

F and E invariant



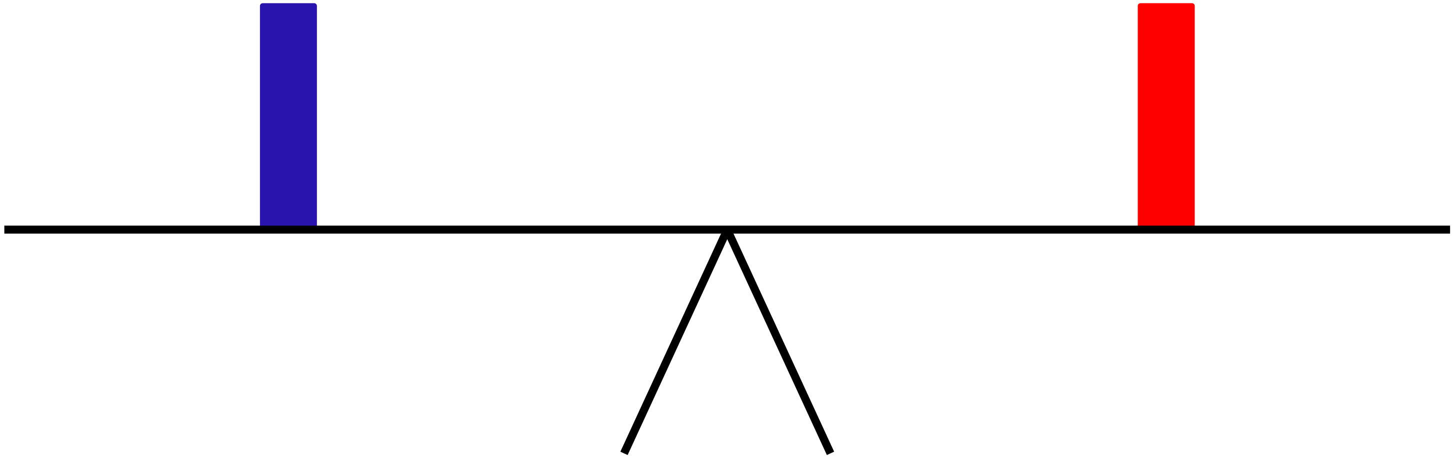
Synoptic scale flow

Conservation laws

$$E = \frac{1}{2}\overline{\mathbf{V}\cdot\mathbf{V}}$$ **Energy**

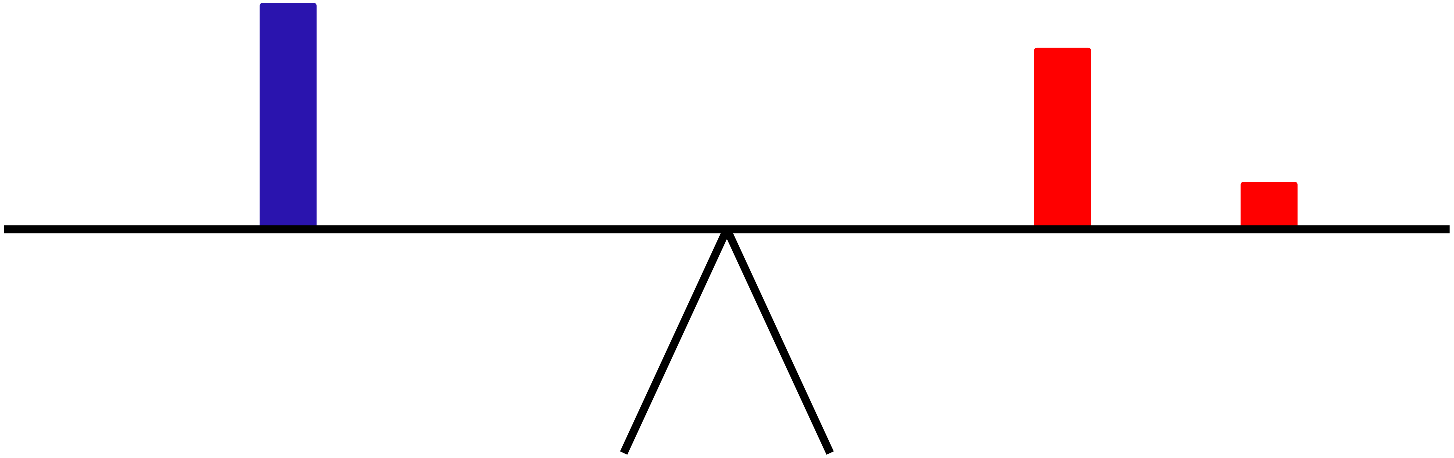
$$F = \frac{1}{2}\overline{\zeta\cdot\zeta}$$ **Enstrophy**

Energy and Enstrophy Conservation



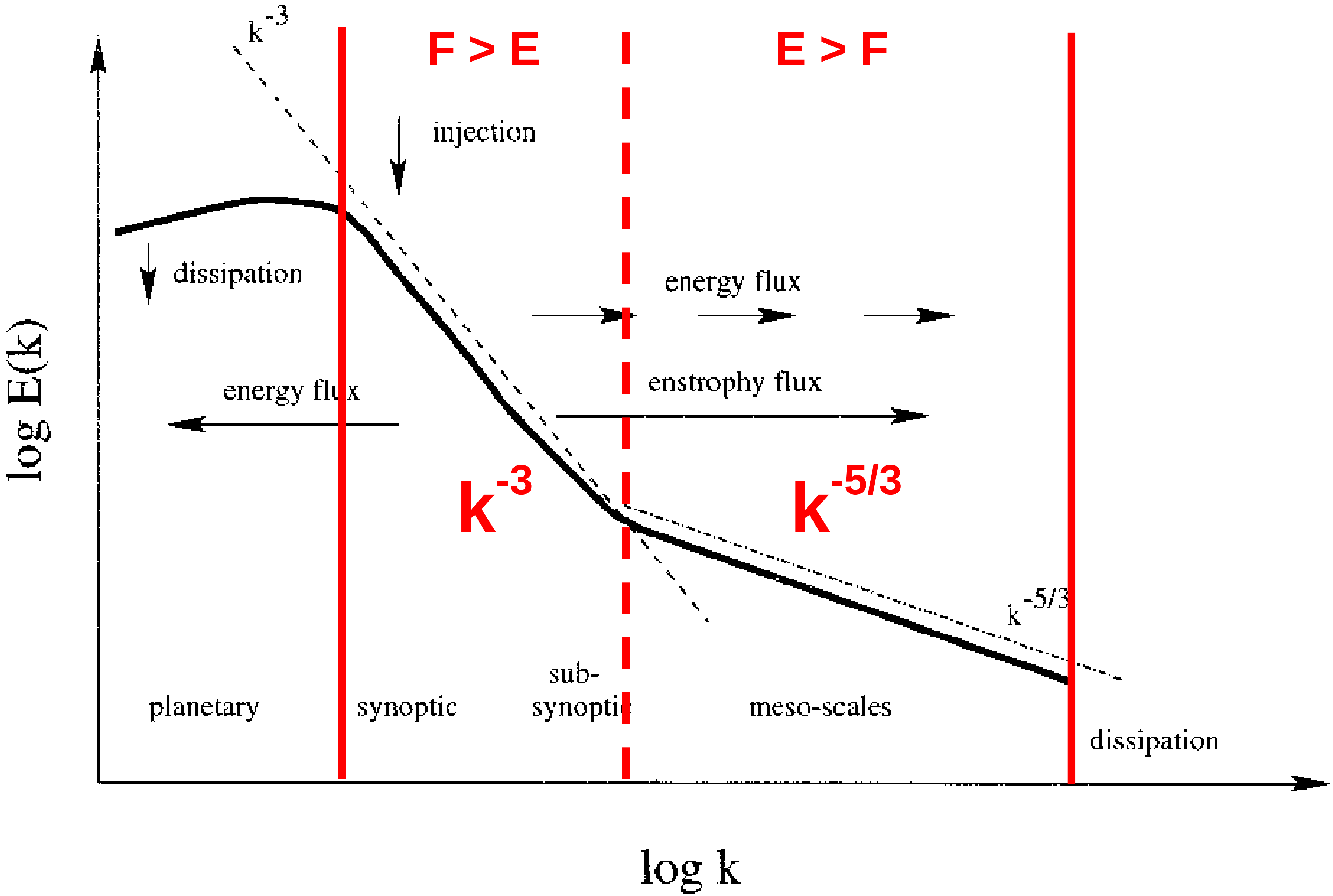
Mechanical analogy

Energy and Enstrophy Conservation

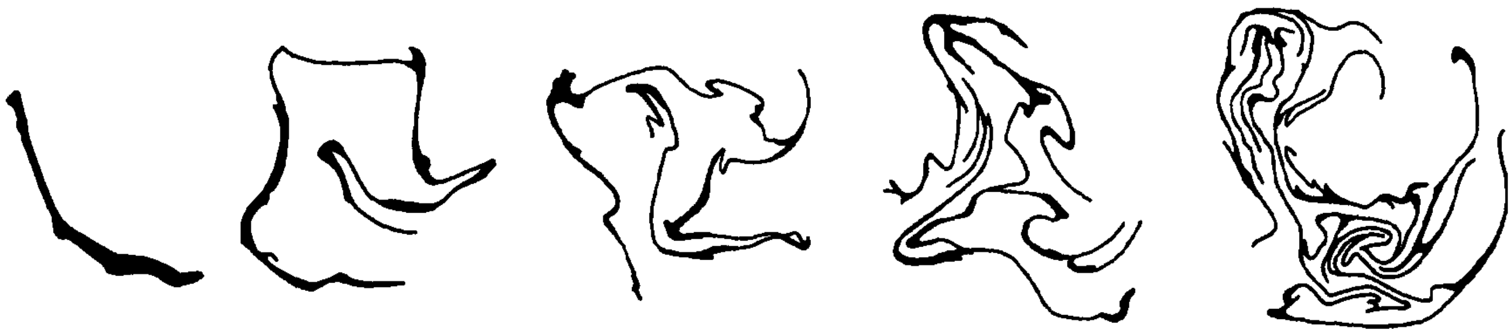


Mechanical analogy

F and E invariant



Enstrophy Cascade



- Moves to smaller scale of motion
- Through differential advection
- Finally destroyed by microscale turbulence

Energy Cascade

- Moves to smaller scale of motion
- Through vortex stretching
- Finally destroyed by microscale turbulence

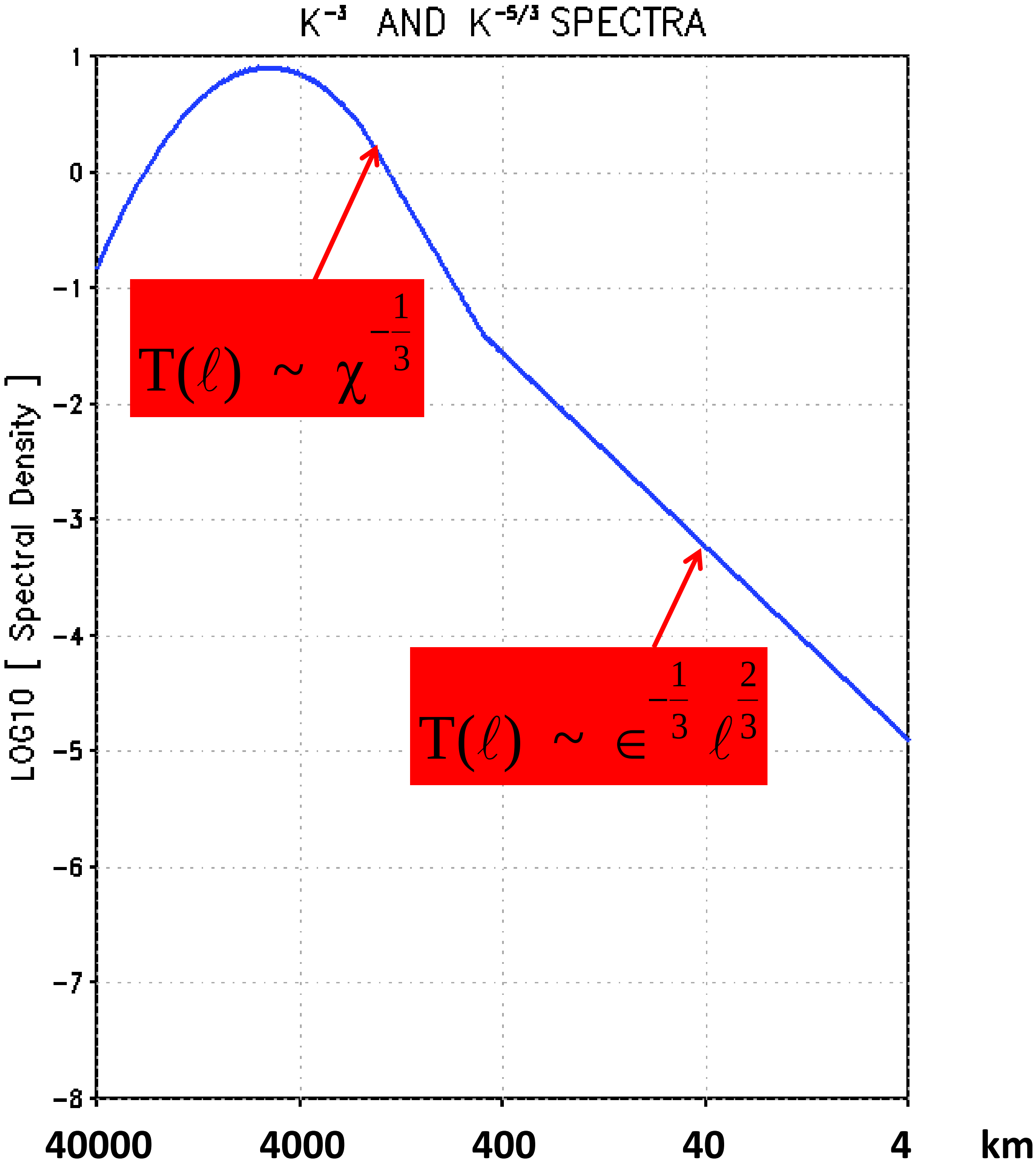
Big whirls have little whirls
That feed on their velocity
And little whirls have lesser whirls
And so on to viscosity

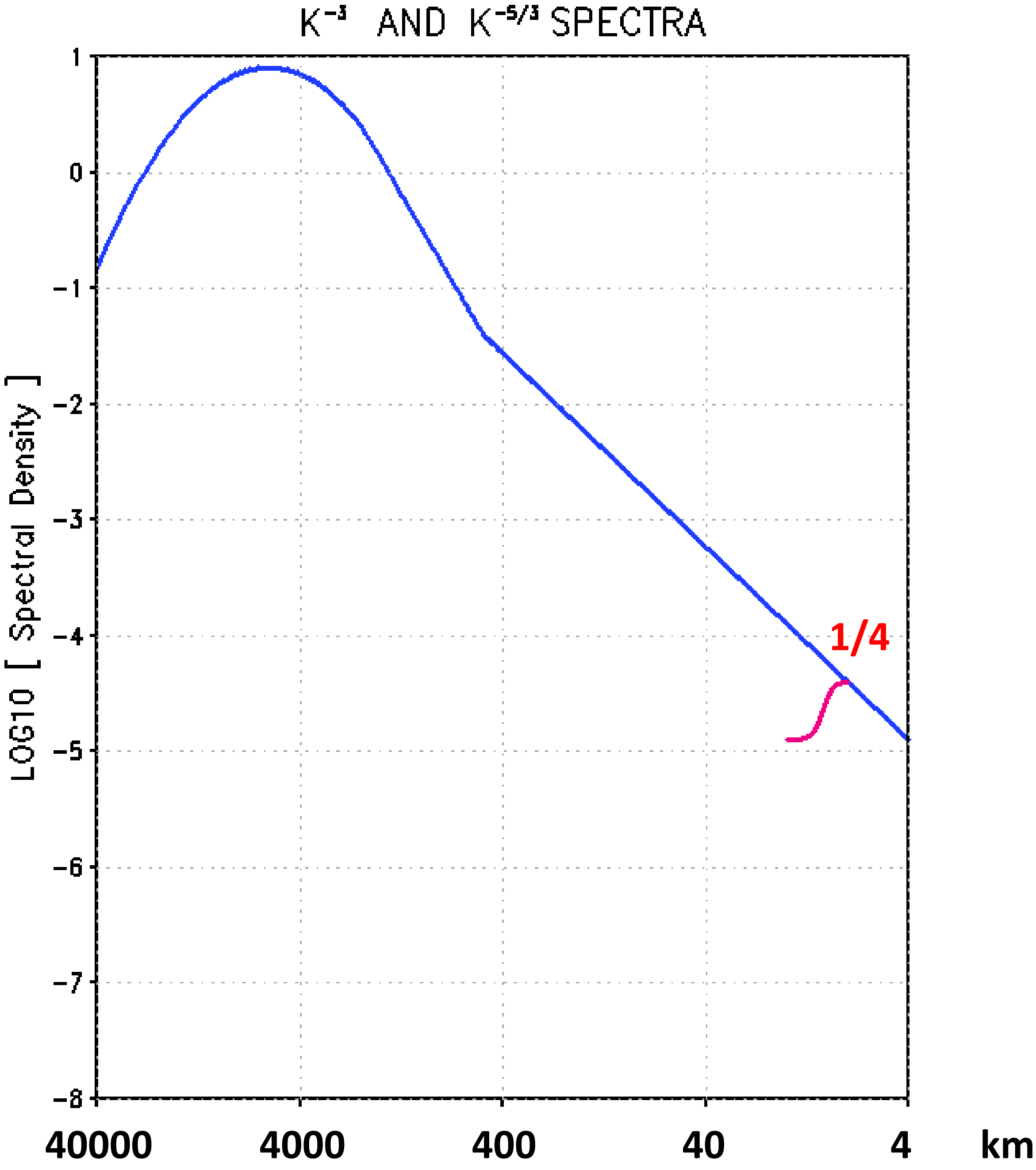
Lewis Fry Richardson

Cascade rates are intimately connected with

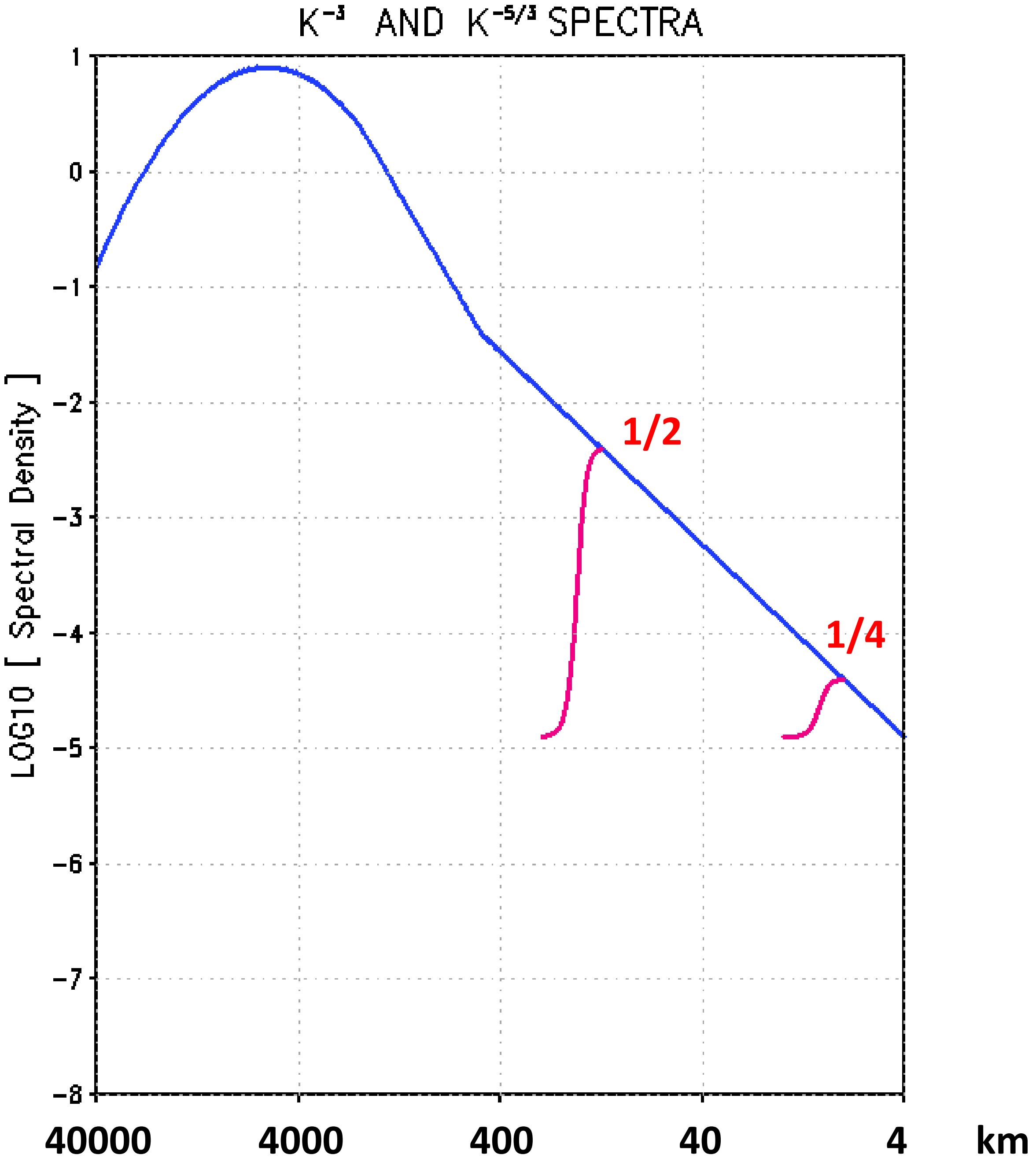
(i) **Eddy turnover times in a turbulent fluid**

(ii) **Spectral slope of the energy spectrum**

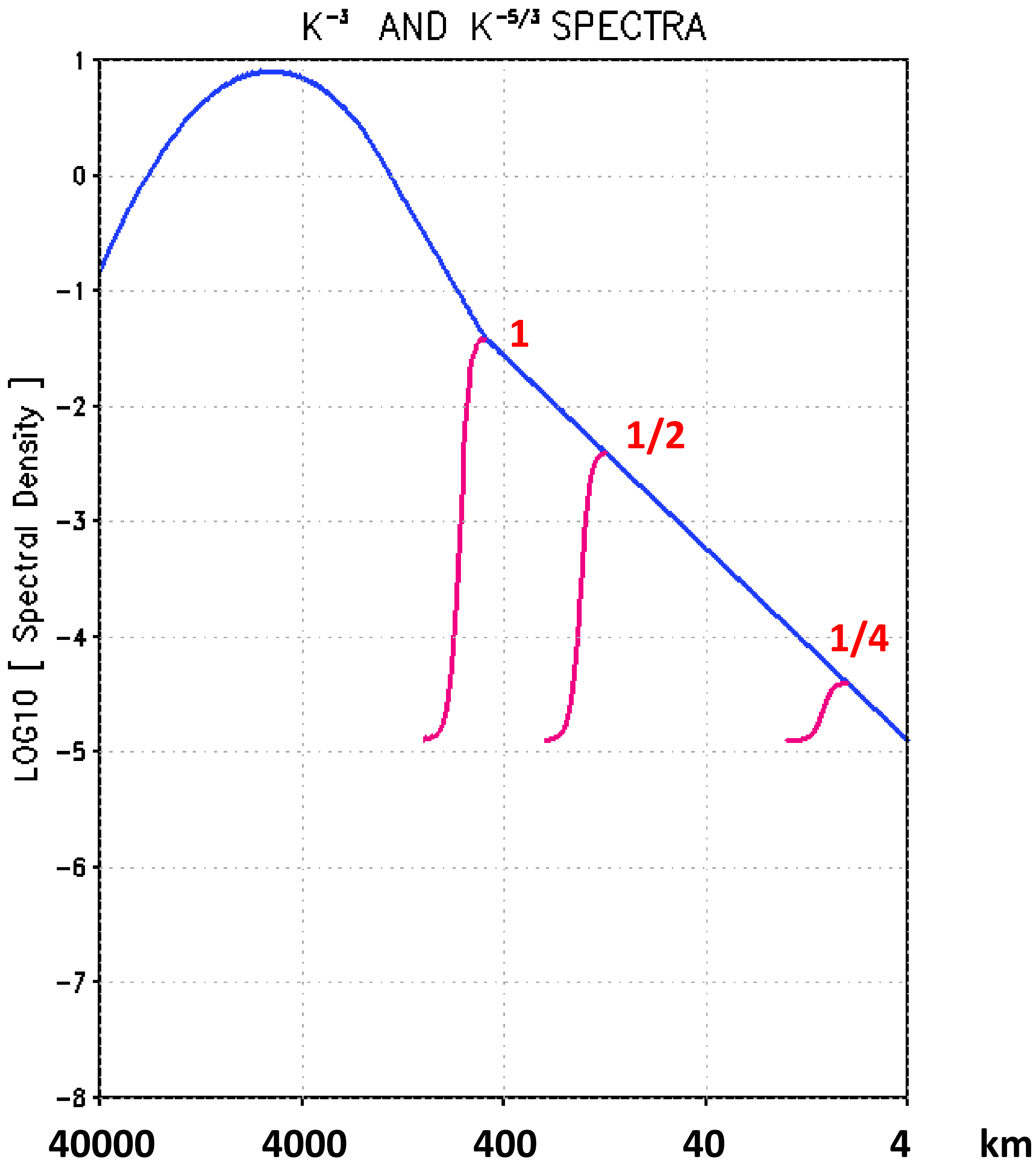

K⁻³ AND K⁻⁵/³ SPECTRA
LOG10 [Spectral Density]
1
0
−1
−2
−3
−4
−5
−6
−7
−8
$T(\ell) \sim \chi^{-\frac{1}{3}}$
$T(\ell) \sim \epsilon^{-\frac{1}{3}} \ell^{\frac{2}{3}}$
40000
4000
400
40
4
km



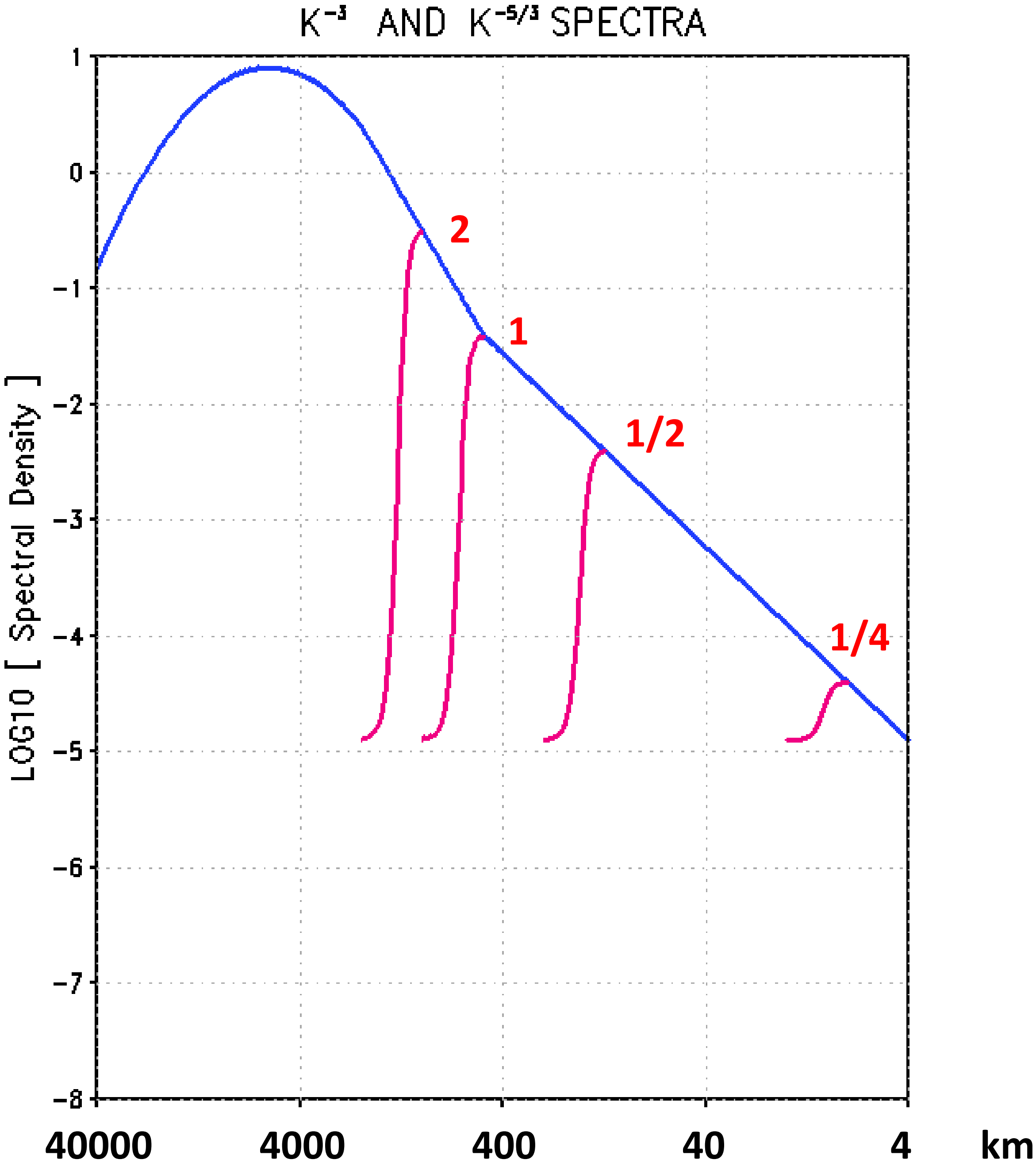
K^-3 AND K^-5/3 SPECTRA
LOG10 [Spectral Density]
1
0
-1
-2
-3
-4
-5
-6
-7
-8
1/4
40000
4000
400
40
4
km



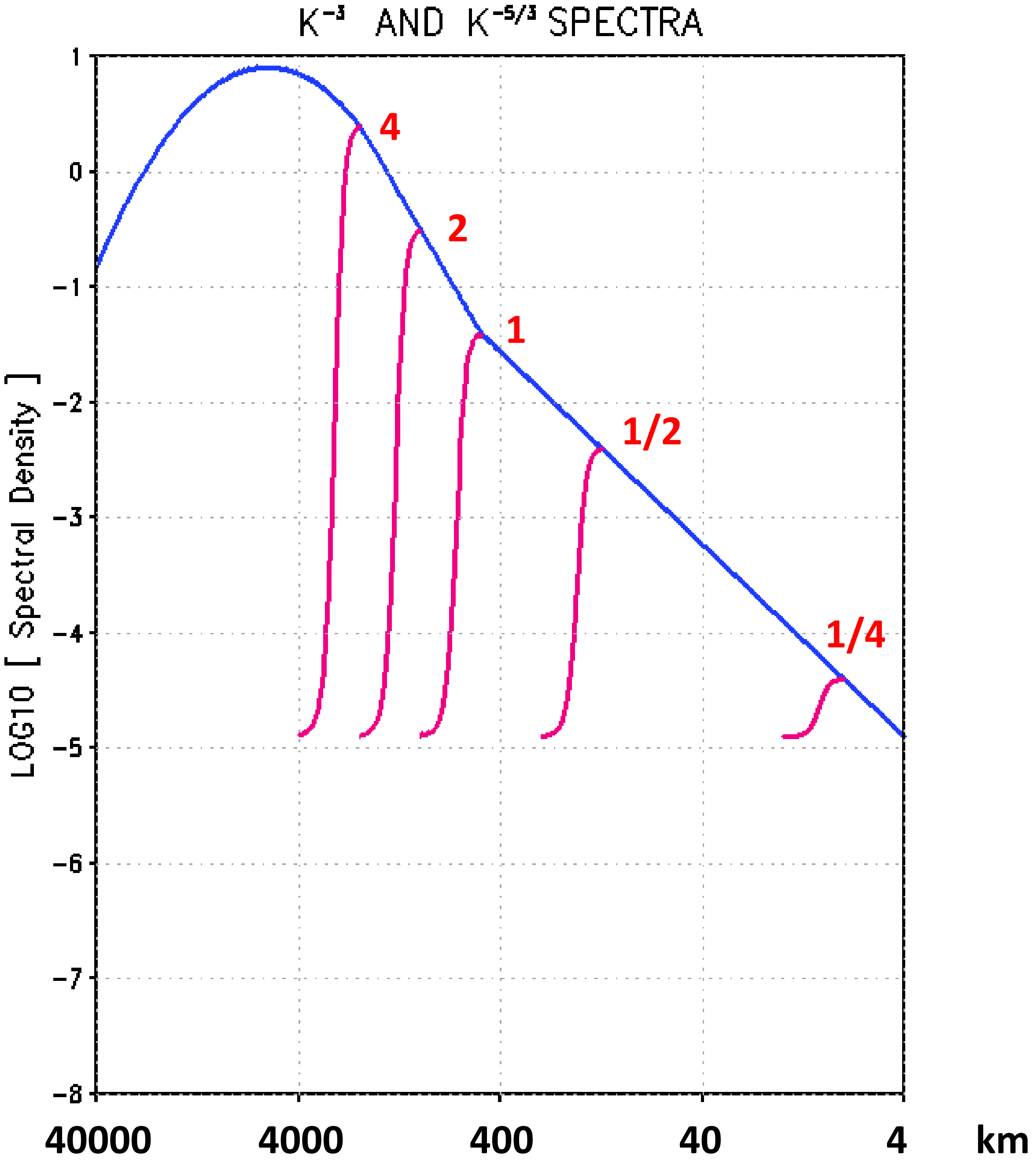
K^{-3} AND $K^{-5/3}$ SPECTRA
LOG10 [Spectral Density]
1
0
−1
−2
−3
−4
−5
−6
−7
−8
1/2
1/4
40000
4000
400
40
4
km

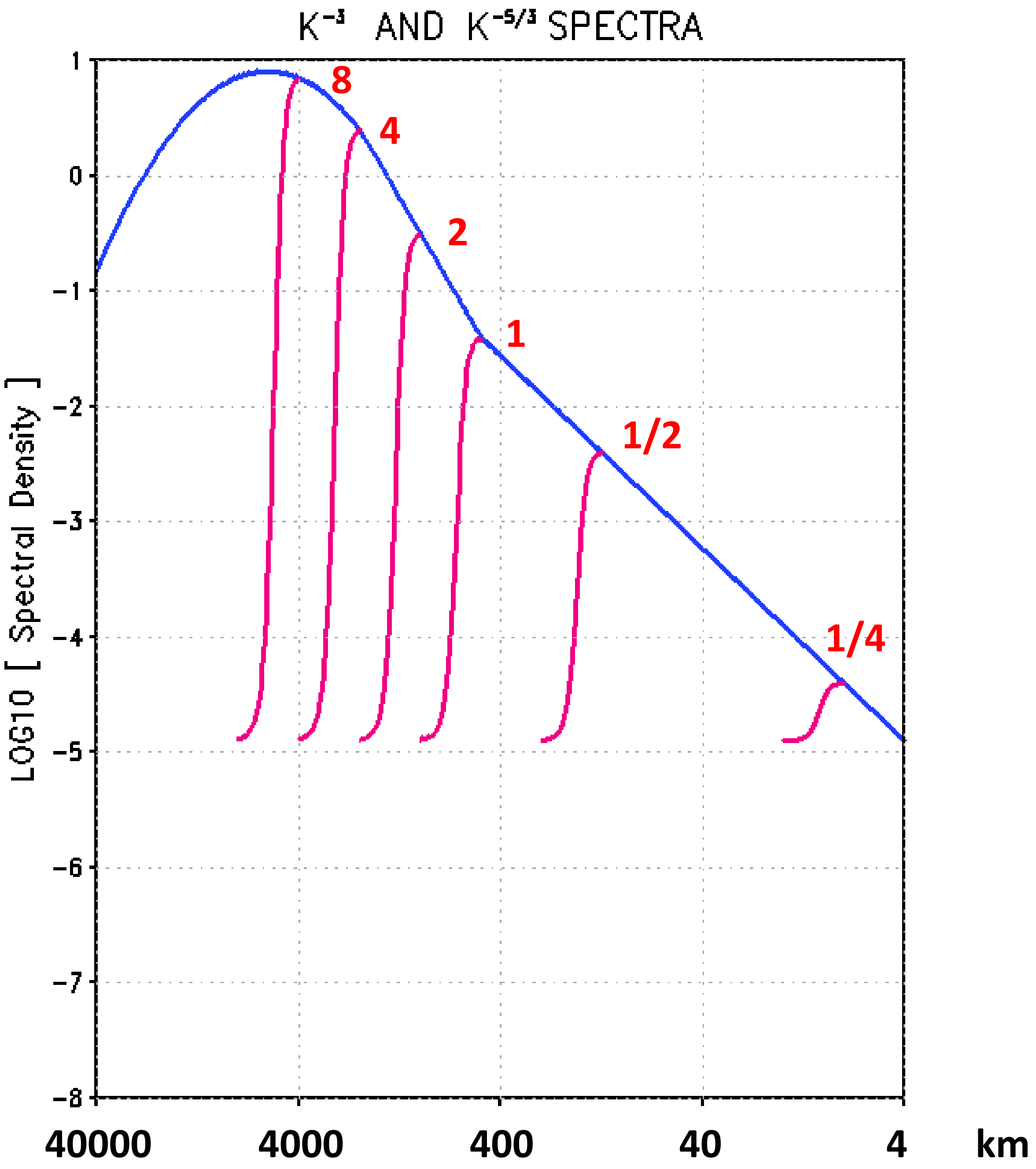


K^{-3} AND K$^{-5/3}$ SPECTRA
LOG10 [Spectral Density]
1
0
−1
−2
−3
−4
−5
−6
−7
−8
1
1/2
1/4
40000
4000
400
40
4
km

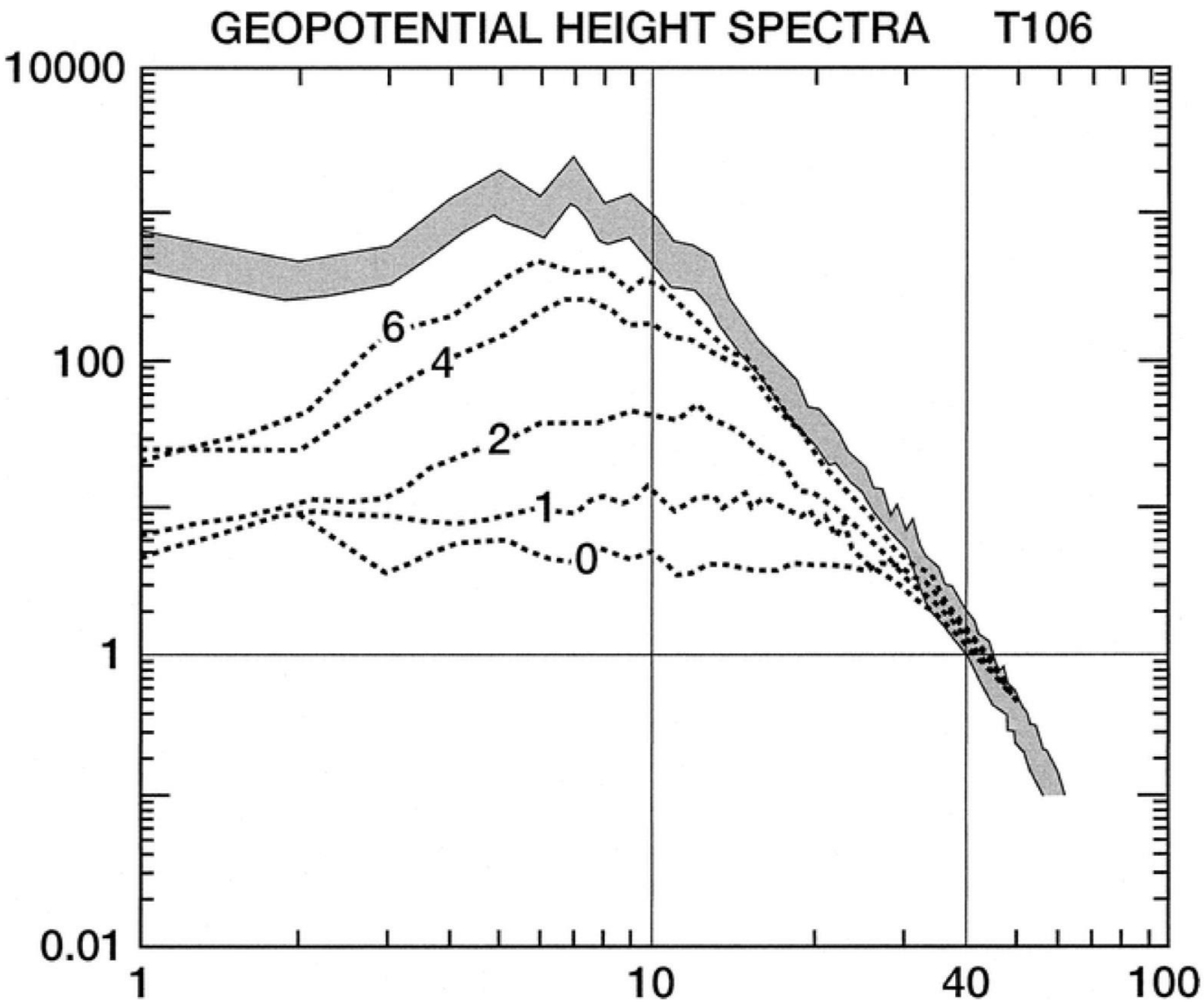


K^{-3} AND $K^{-5/3}$ SPECTRA
LOG10 [Spectral Density]
1
0
-1
-2
-3
-4
-5
-6
-7
-8
2
1
1/2
1/4
40000
4000
400
40
4
km


K⁻³ AND K⁻⁵/³ SPECTRA
LOG10 [Spectral Density]
1
0
−1
−2
−3
−4
−5
−6
−7
−8
4
2
1
1/2
1/4
40000
4000
400
40
4
km


K^-3 AND K^-5/3 SPECTRA
LOG10 [Spectral Density]
1
0
-1
-2
-3
-4
-5
-6
-7
-8
8
4
2
1
1/2
1/4
40000
4000
400
40
4
km

Realistic Error Spectrum



Tribbia and Baumhefner 2004

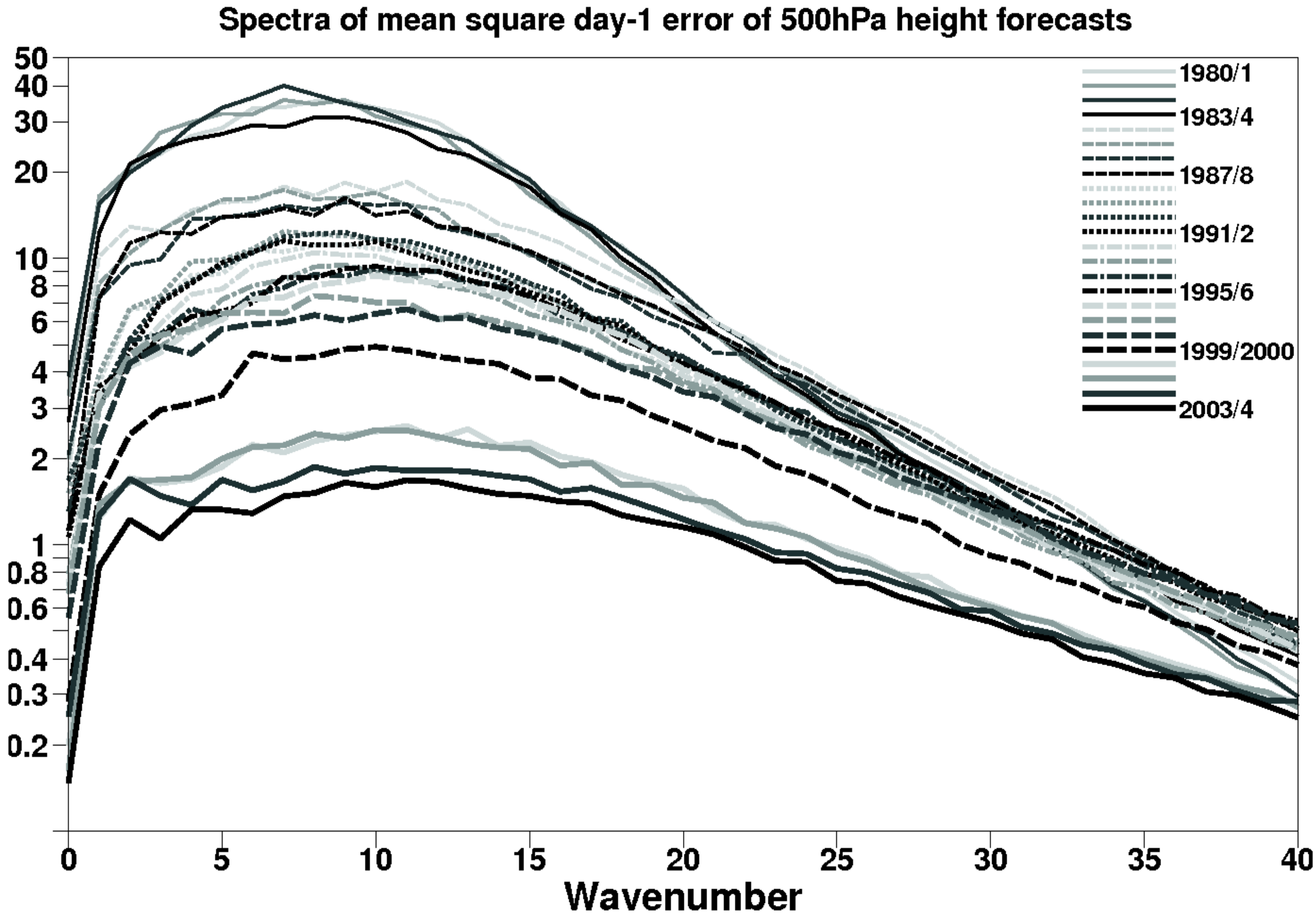
Three types of Error Growth

1. Inverse cascade

2. Baroclinic instability

3. Advection

Observed Improvements in Initial Error



Simmons 2006

1. Errors are initially present on all scales

2. Errors in small scales saturate very fast

3. Errors in the synoptic scales grow primarily due to instabilities, not due to inverse cascade process

4. The predictable part of the flow beyond a few hours are mostly in the synoptic scales

1. Errors are initially present on all scales

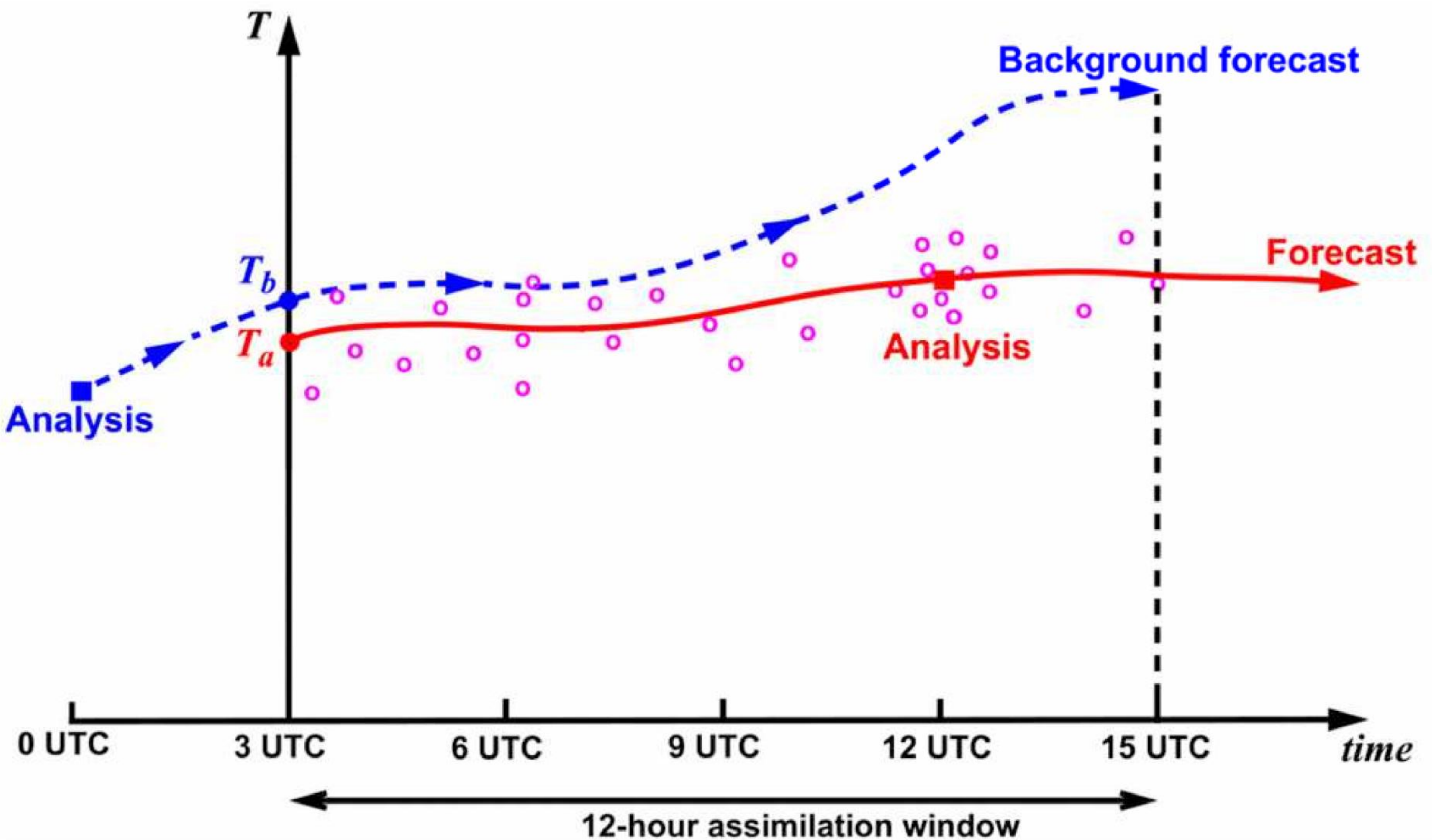
2. Errors in small scales saturate very fast

3. Errors in the synoptic scales grow primarily due to instabilities, not due to inverse cascade process

4. The predictable part of the flow beyond a few hours are mostly in the synoptic scales

Data Assimilation

Traditional 4DVAR



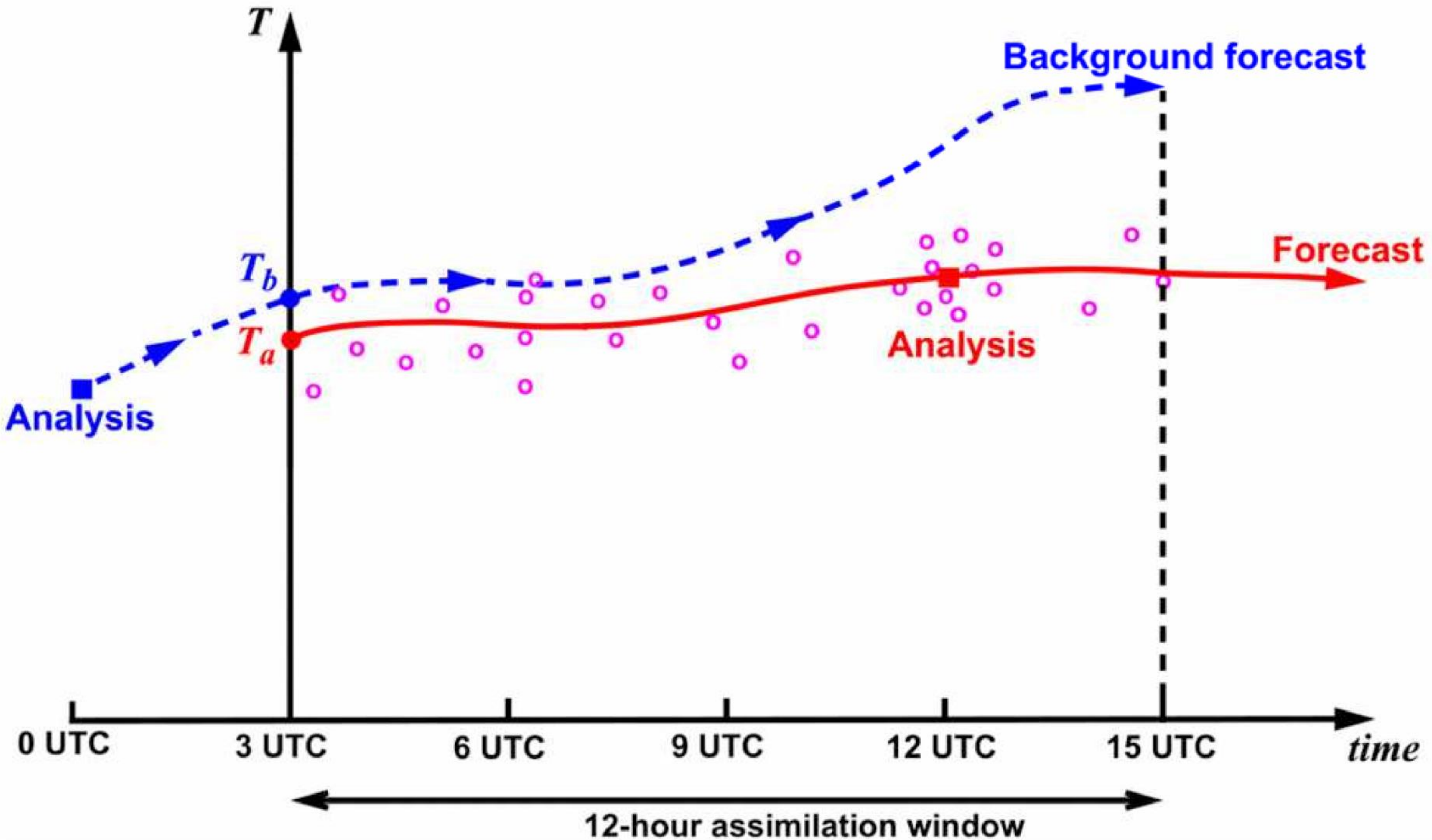
Quantity of available data

$$\text{Number of Observations} = M \sim 10^5 - 10^6$$

$$\text{Dimension of State Vector} = N \sim 10^7 - 10^9$$

M << N

$$J(\mathbf{x}_o) = J_b + J_o = \frac{1}{2}\left(\mathbf{x}_o - \mathbf{x}_o^b\right)^T \mathbf{B}^{-1}\left(\mathbf{x}_o - \mathbf{x}_o^b\right) + \sum_{i=1}^{I} \frac{1}{2}\left(H_i \mathbf{x}_i - \mathbf{y}_i\right)^T \mathbf{R}_i^{-1}\left(H_i \mathbf{x}_i - \mathbf{y}_i\right)$$

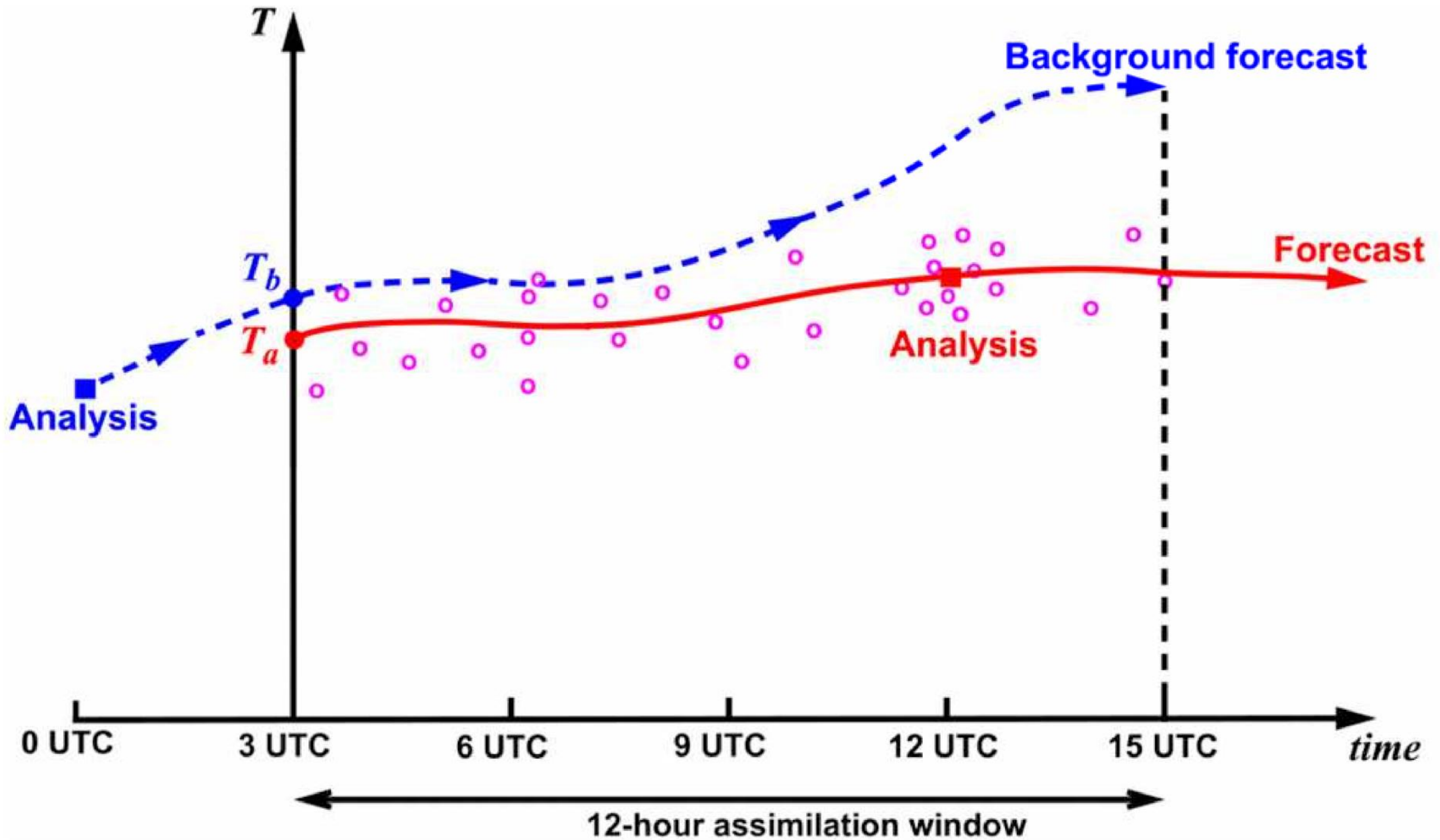


Data Assimilation

↕

Ensembles

$$J(\mathbf{x}_o) = J_b + J_o = \frac{1}{2}\left(\mathbf{x}_o - \mathbf{x}_\mathbf{o}^\mathbf{b}\right)^T \mathbf{B}^{-1}\left(\mathbf{x}_o - \mathbf{x}_\mathbf{o}^\mathbf{b}\right) + \sum_{i=1}^{I}\frac{1}{2}\left(H_\mathbf{i}\,\mathbf{x}_i - \mathbf{y}_\mathbf{i}\right)^T \mathbf{R}_i^{-1}\left(H_\mathbf{i}\,\mathbf{x}_i - \mathbf{y}_\mathbf{i}\right)$$



Quantity of available data

$$\text{Number of Observations} = M \sim 10^5 - 10^6$$

$$\text{Dimension of State Vector} = N \sim 10^7 - 10^9$$

M << N

⬇

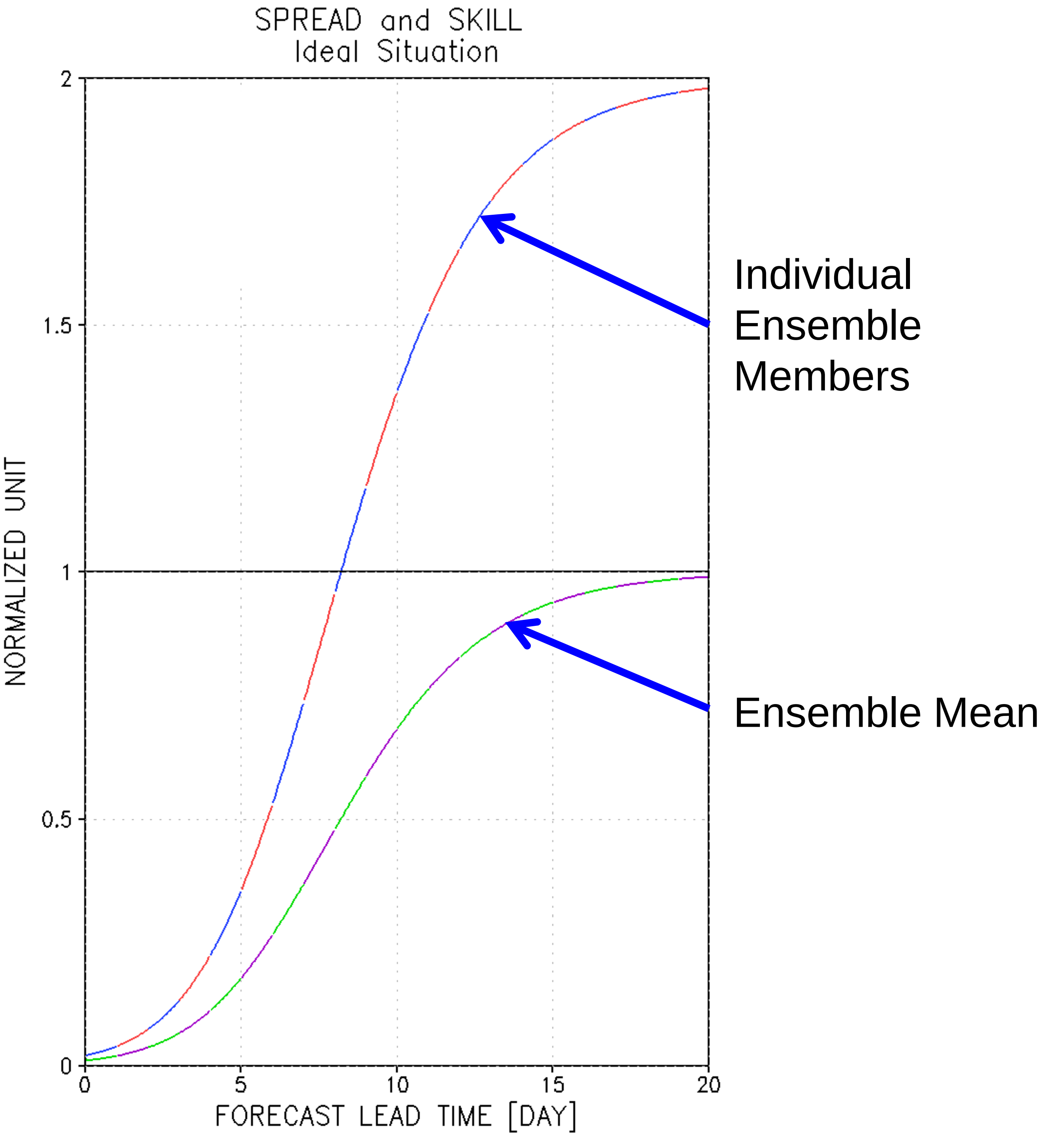
Only the largest scales are really defined by the available data

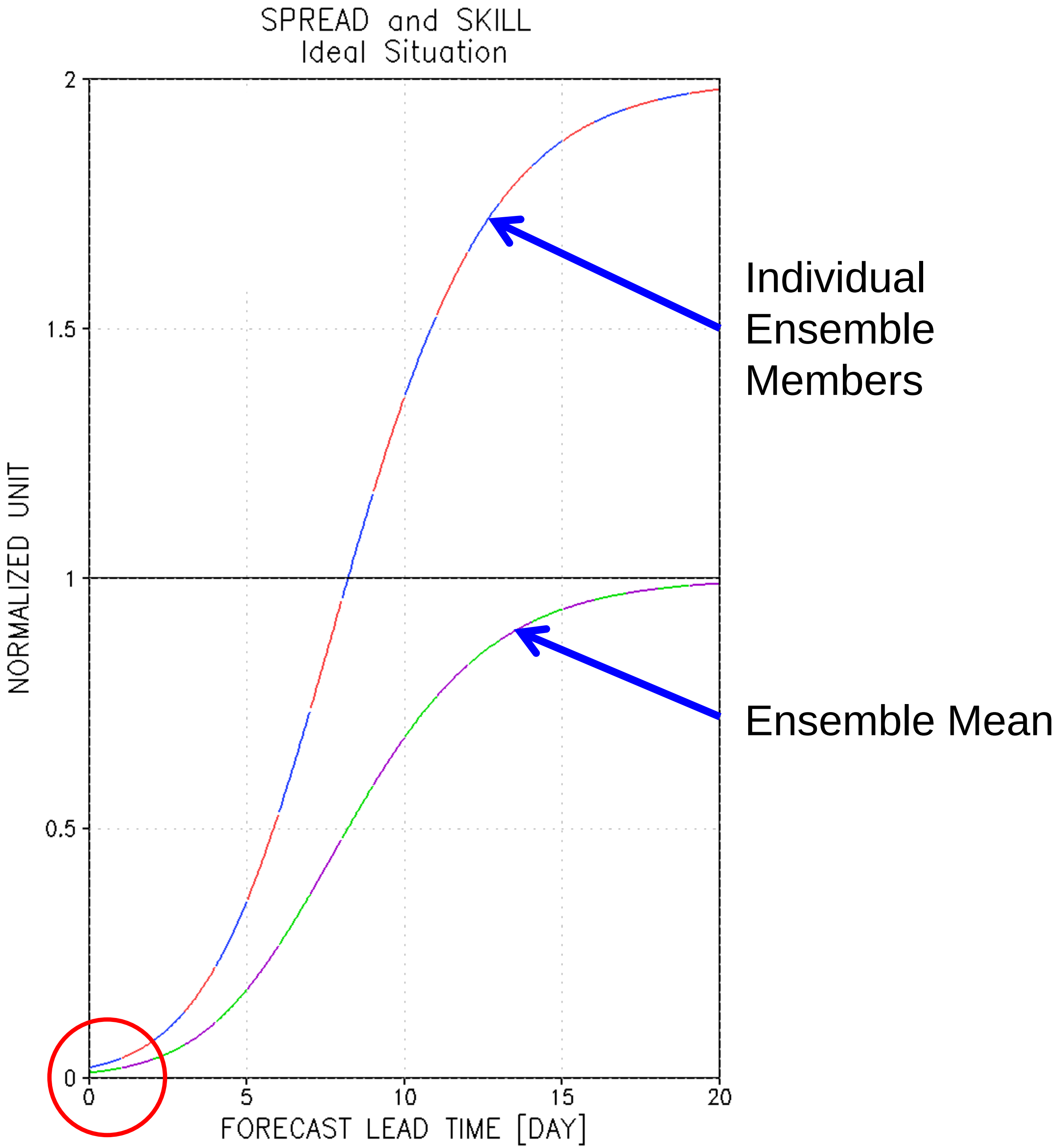
Smaller scales are not

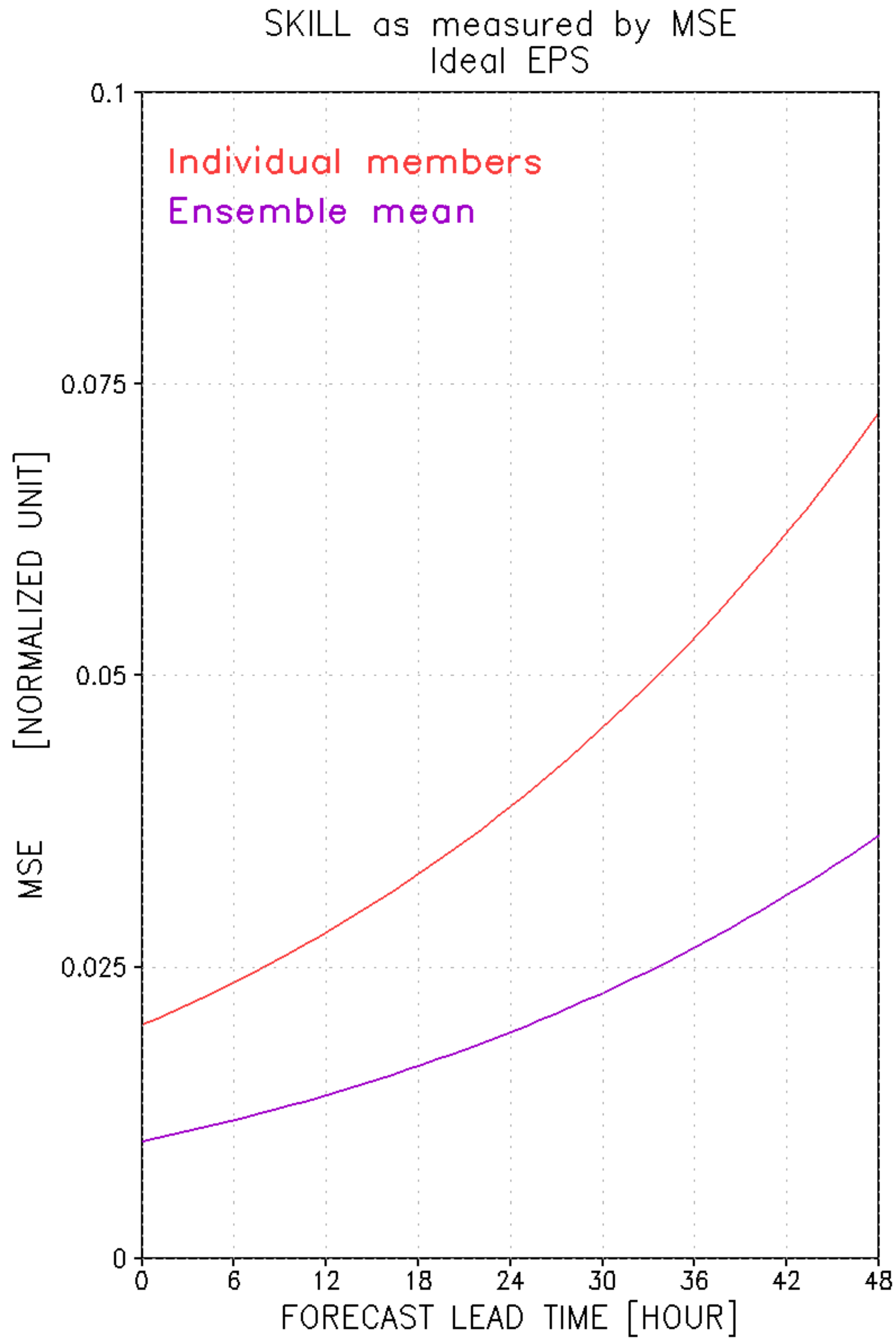
There should thus be an infinite amount of equally likely IC

↔

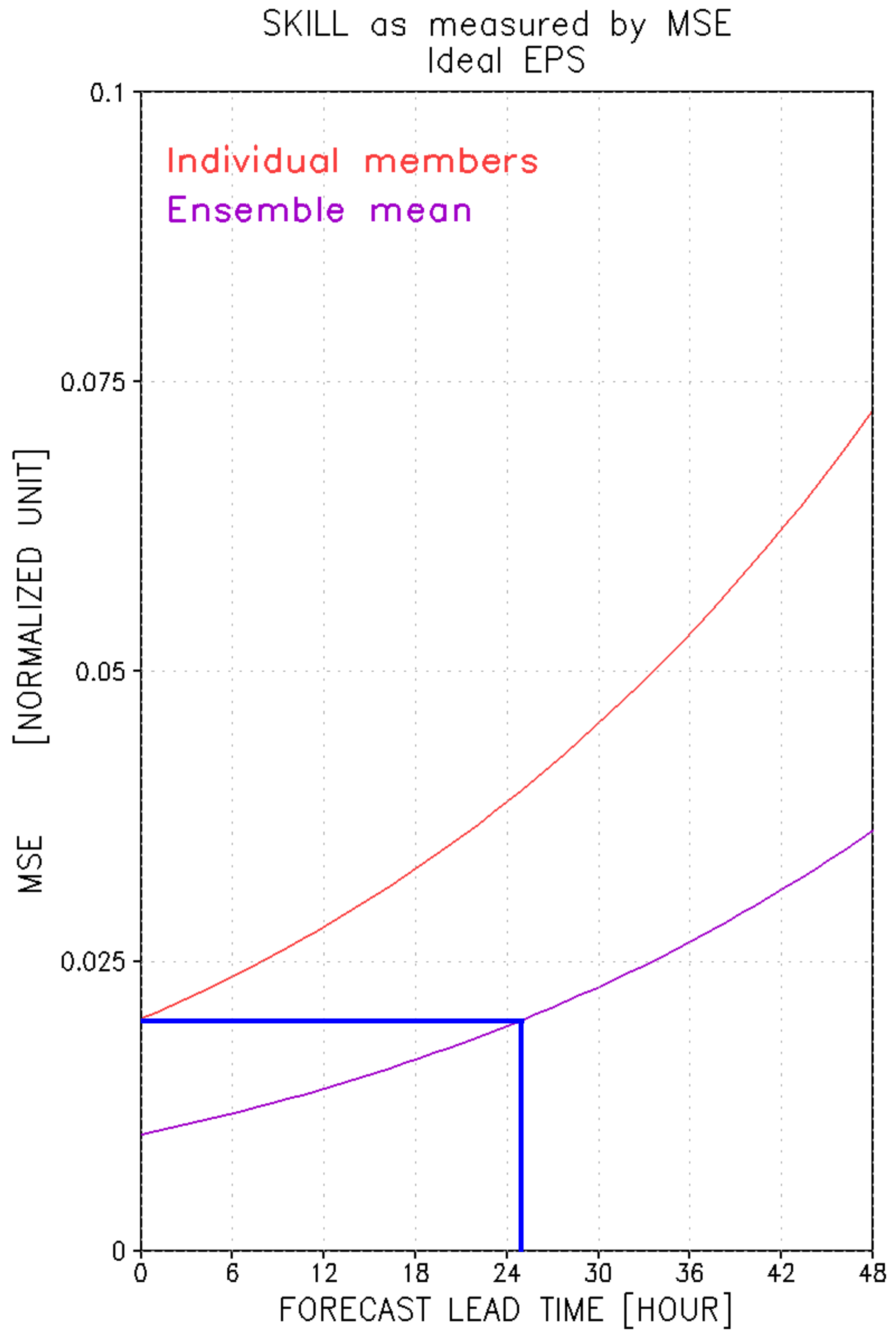
Ensembles of IC

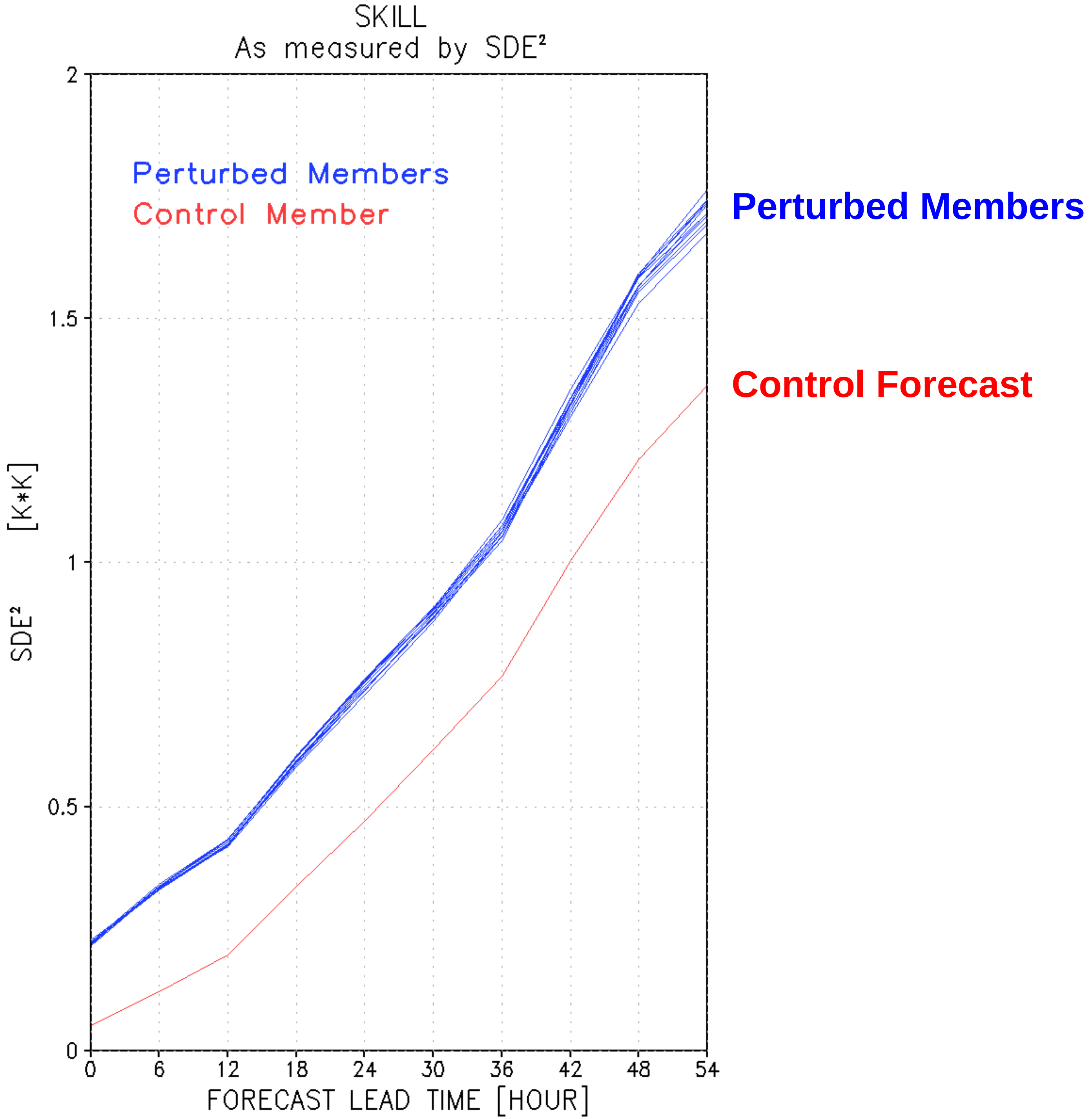

SPREAD and SKILL
Ideal Situation
NORMALIZED UNIT
FORECAST LEAD TIME [DAY]
0
0.5
1
1.5
2
0
5
10
15
20
Individual Ensemble Members
Ensemble Mean

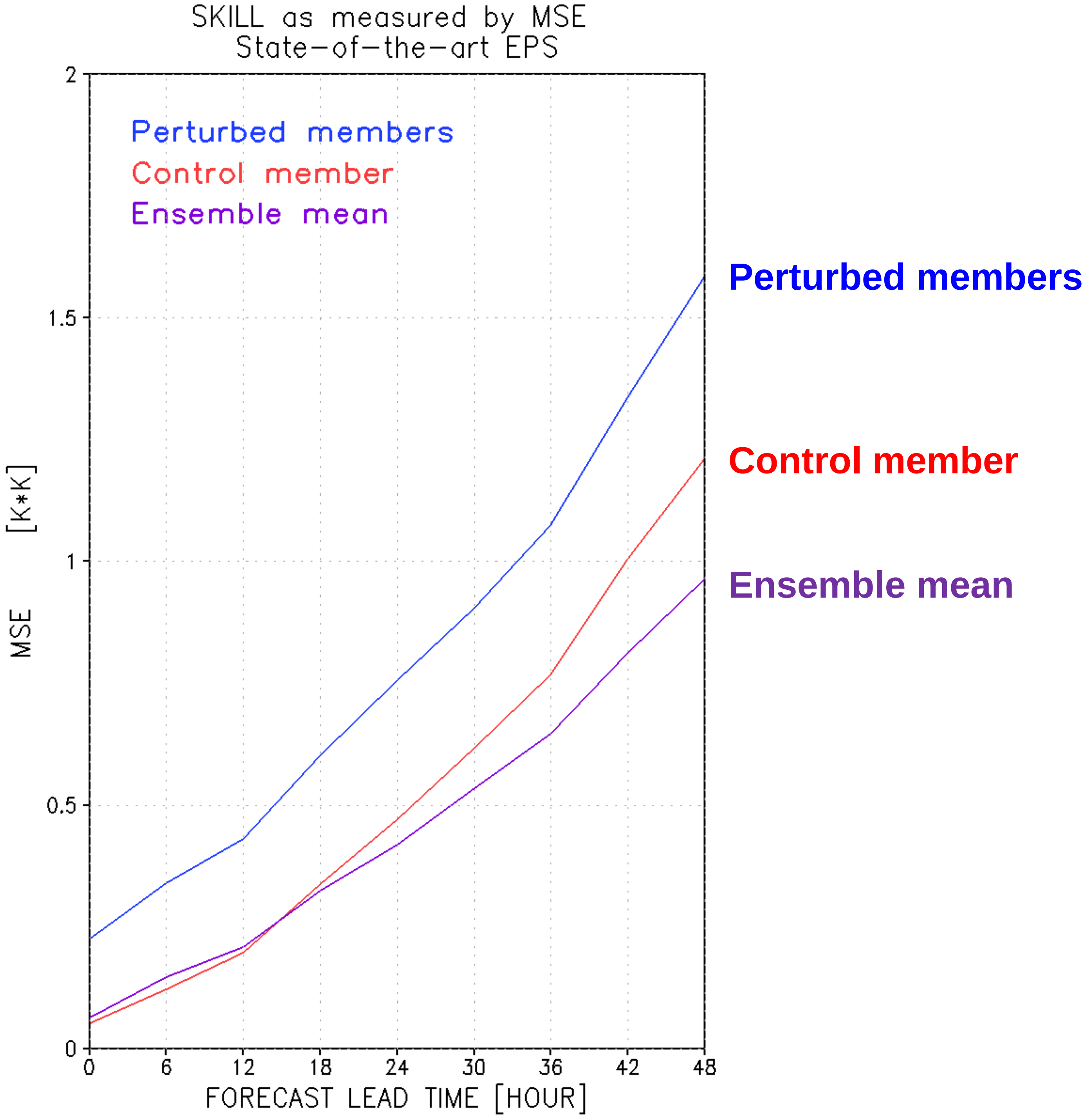

SPREAD and SKILL
Ideal Situation
NORMALIZED UNIT
FORECAST LEAD TIME [DAY]
2
1.5
1
0.5
0
0
5
10
15
20
Individual Ensemble Members
Ensemble Mean



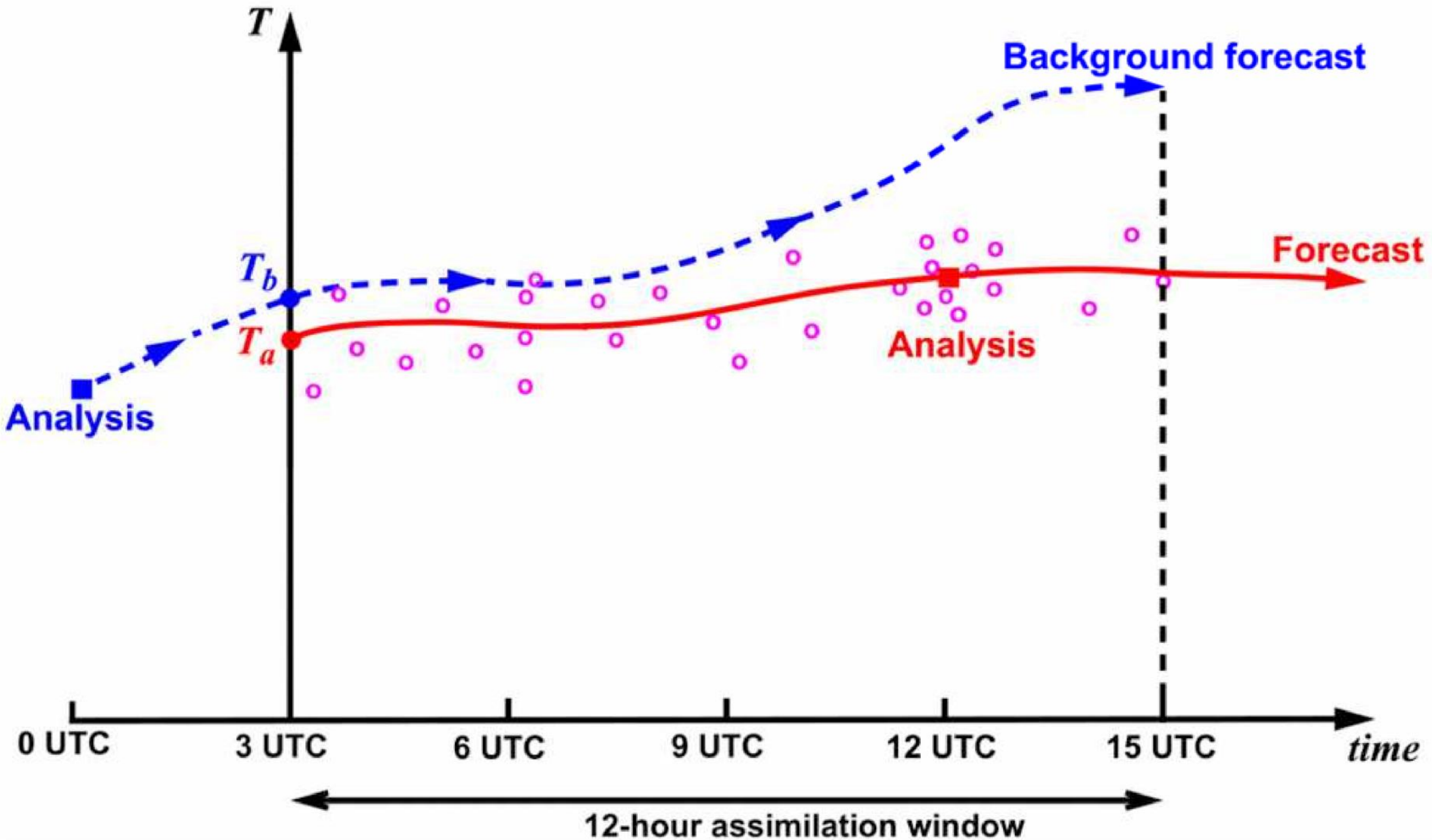
SKILL as measured by MSE
Ideal EPS
Individual members
Ensemble mean
MSE [NORMALIZED UNIT]
0.1
0.075
0.05
0.025
0
0
6
12
18
24
30
36
42
48
FORECAST LEAD TIME [HOUR]


SKILL as measured by MSE
Ideal EPS
Individual members
Ensemble mean
MSE [NORMALIZED UNIT]
0.1
0.075
0.05
0.025
0
0
6
12
18
24
30
36
42
48
FORECAST LEAD TIME [HOUR]

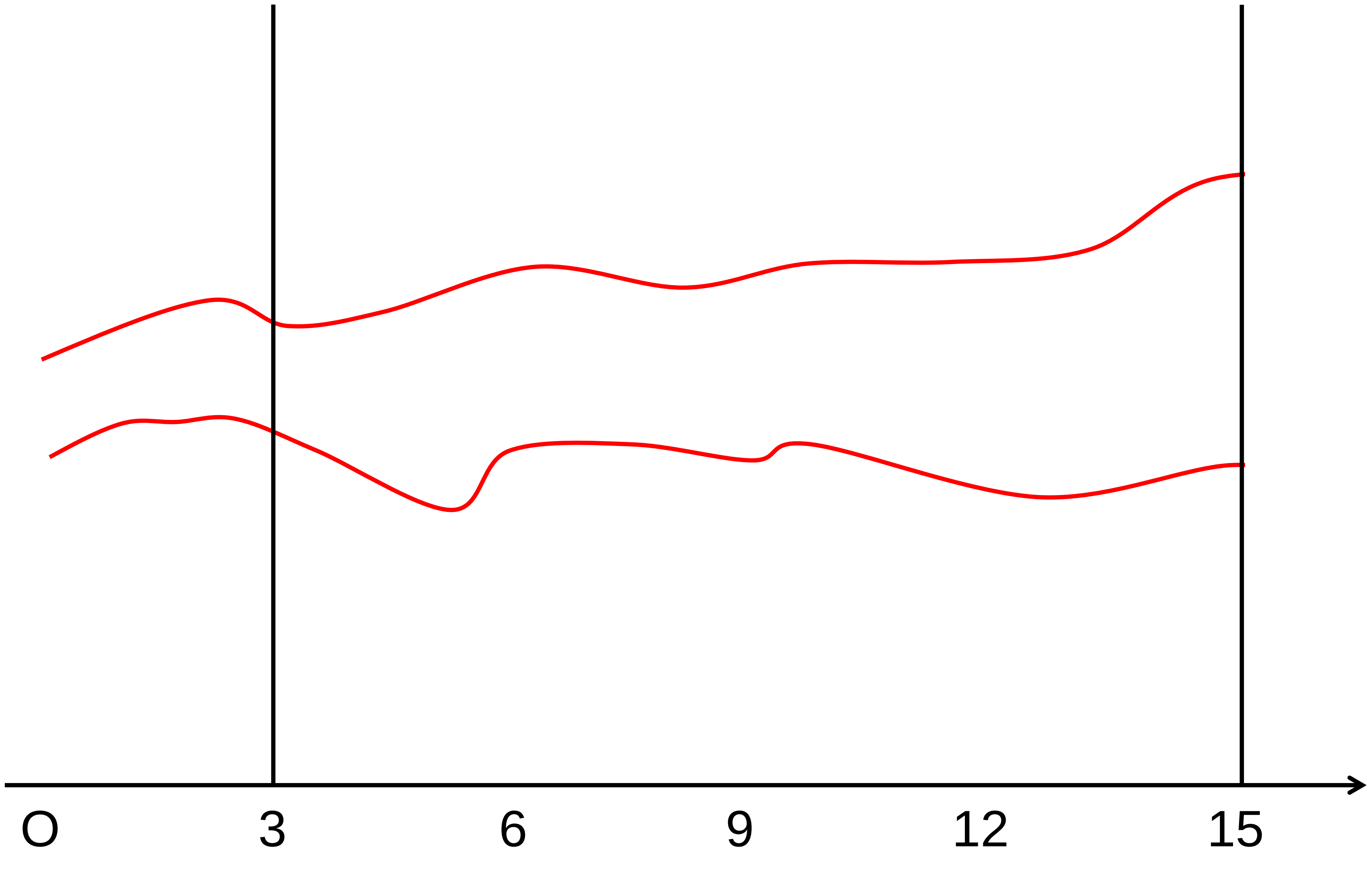

SKILL
As measured by SDE²
Perturbed Members
Control Member
SDE² [K*K]
2
1.5
1
0.5
0
0
6
12
18
24
30
36
42
48
54
FORECAST LEAD TIME [HOUR]
Perturbed Members
Control Forecast


SKILL as measured by MSE
State-of-the-art EPS
Perturbed members
Control member
Ensemble mean
MSE [K*K]
2
1.5
1
0.5
0
0
6
12
18
24
30
36
42
48
FORECAST LEAD TIME [HOUR]
Perturbed members
Control member
Ensemble mean

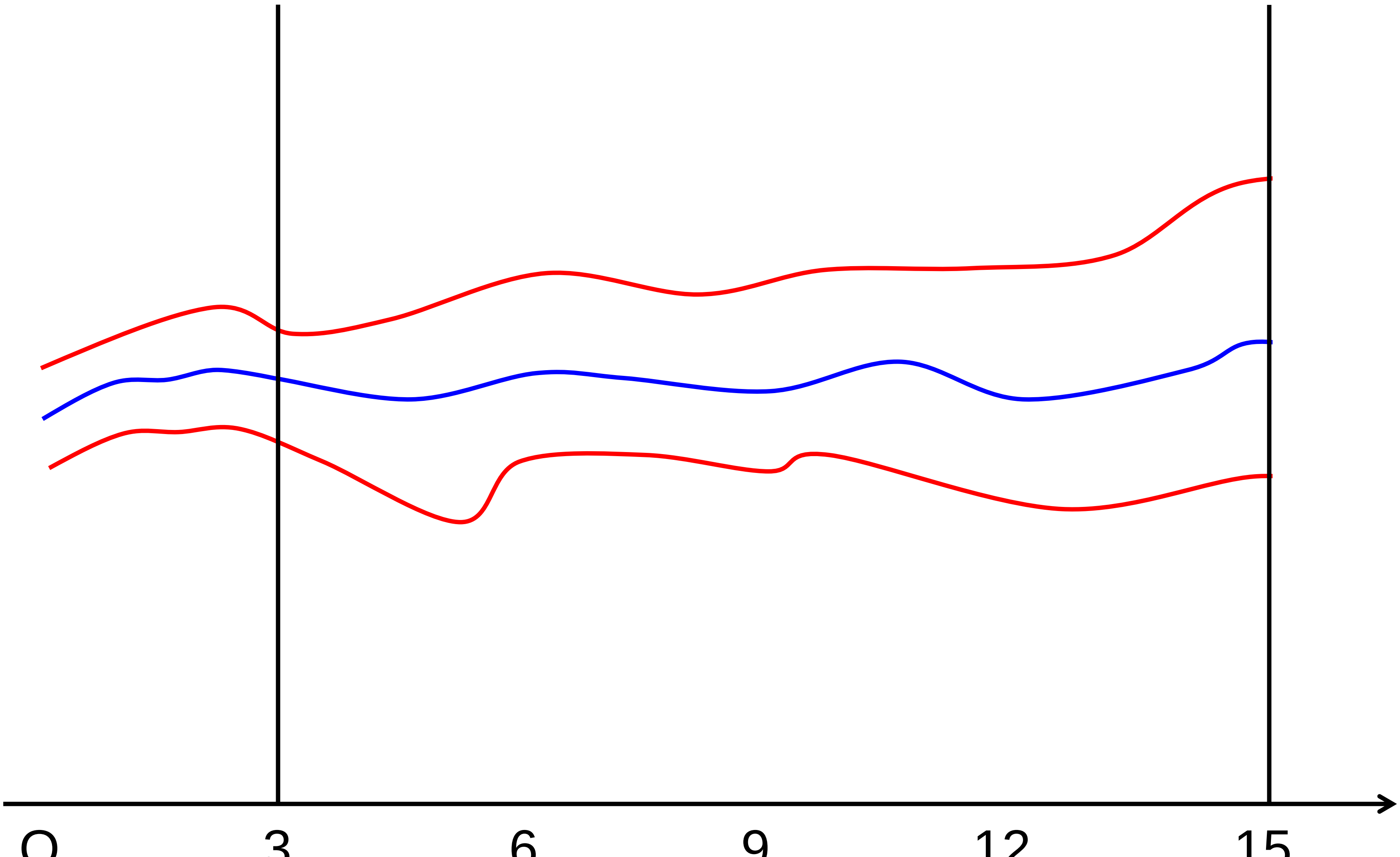
$$J(\mathbf{x}_o) = J_b + J_o = \frac{1}{2}\left(\mathbf{x}_o - \mathbf{x}_\mathbf{o}^\mathbf{b}\right)^T \mathbf{B}^{-1}\left(\mathbf{x}_o - \mathbf{x}_\mathbf{o}^\mathbf{b}\right) + \sum_{i=1}^{I} \frac{1}{2}\left(H_\mathbf{i}\,\mathbf{x}_i - \mathbf{y}_\mathbf{i}\right)^T \mathbf{R}_i^{-1}\left(H_\mathbf{i}\,\mathbf{x}_i - \mathbf{y}_\mathbf{i}\right)$$



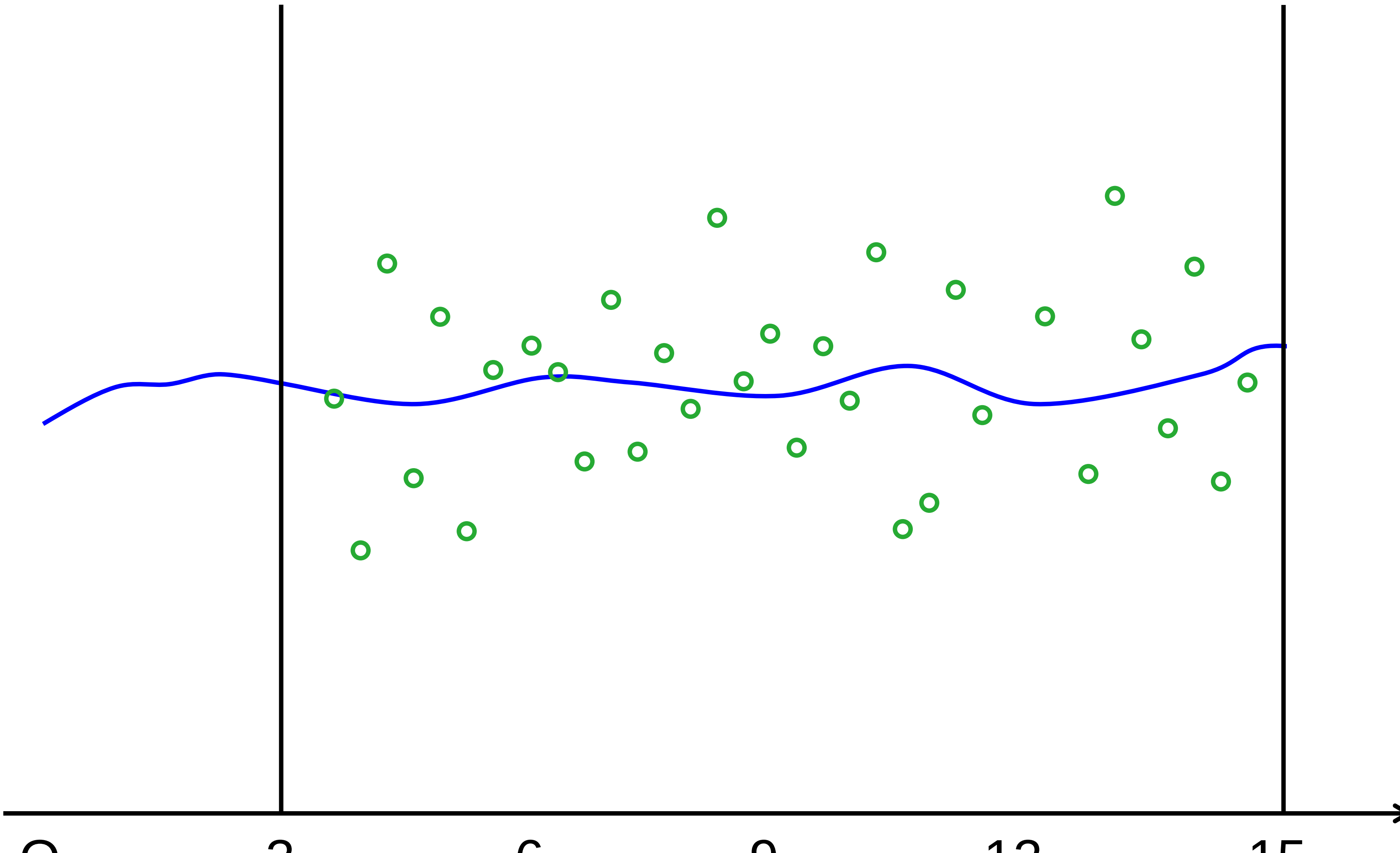
Ensemble Mean 4DVAR



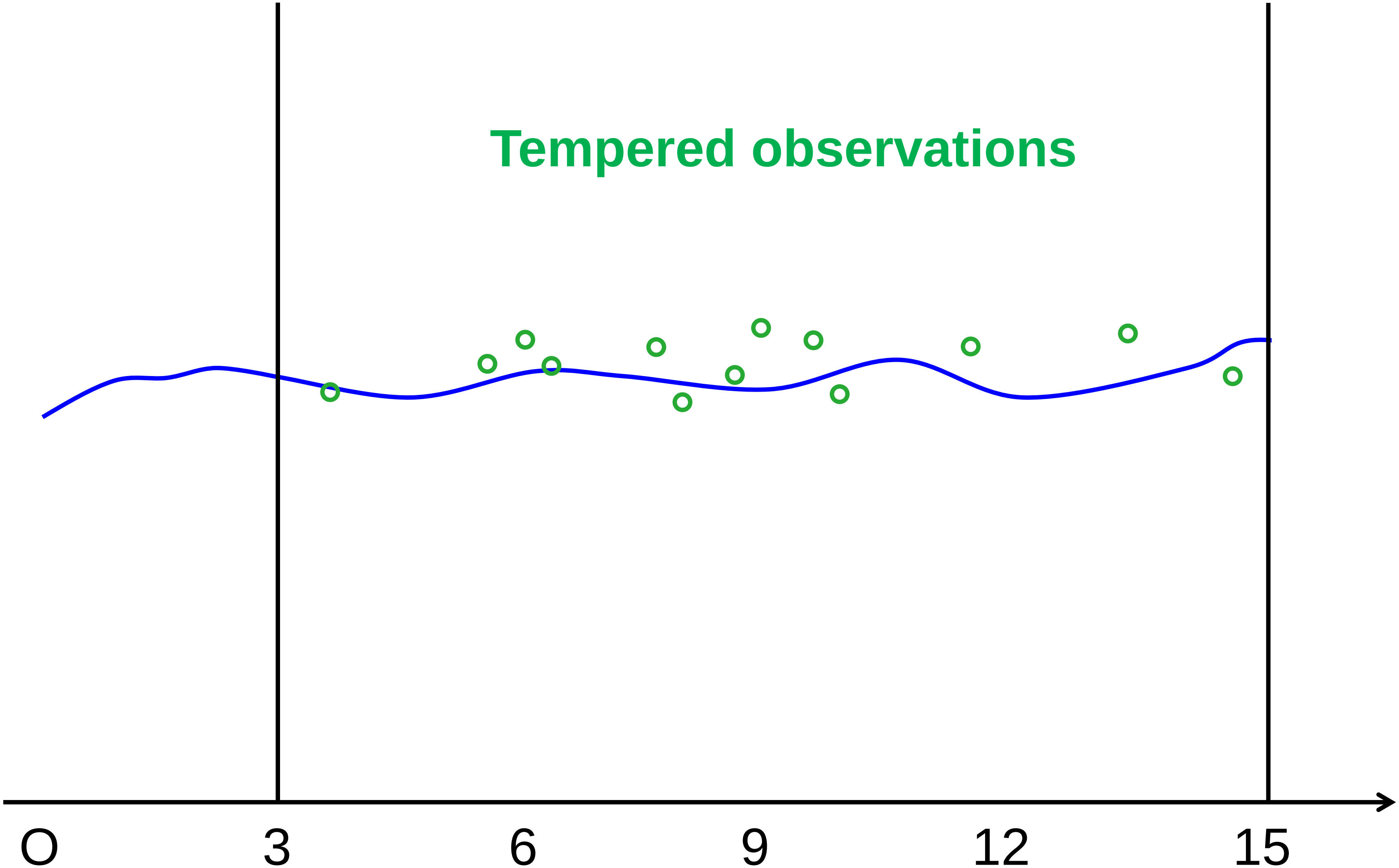
Ensemble Mean 4DVAR



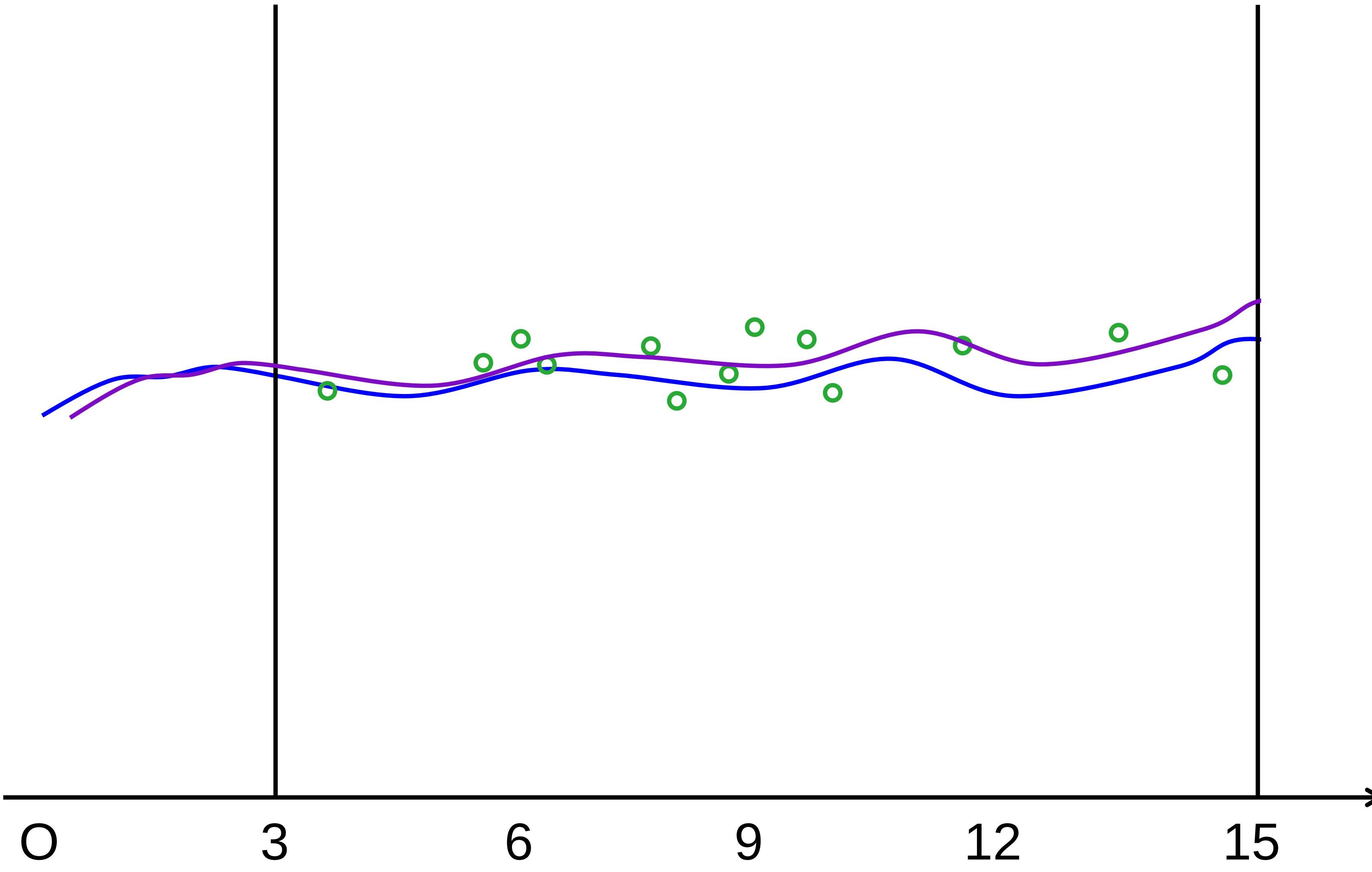
Ensemble Mean 4DVAR



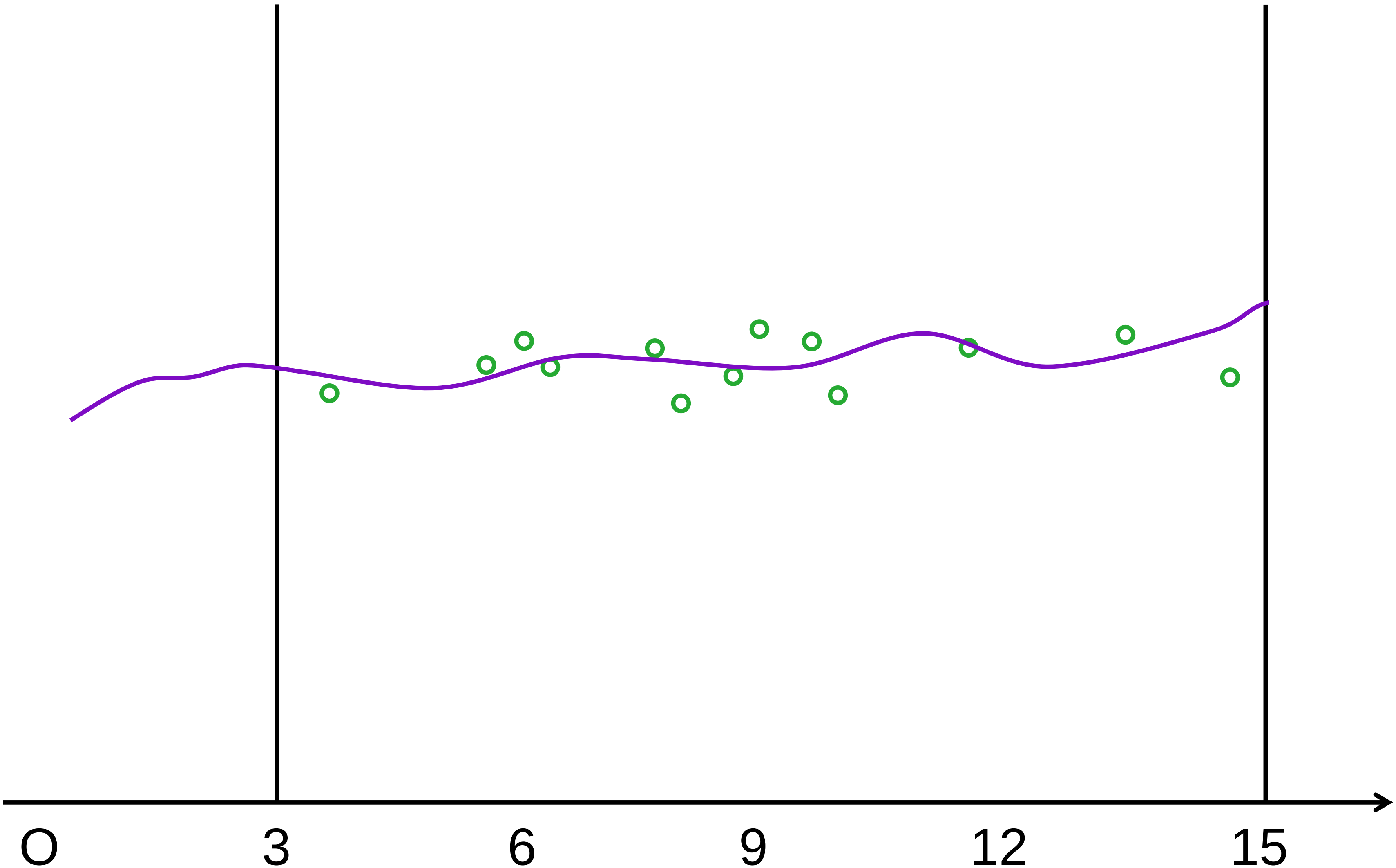
Ensemble Mean 4DVAR



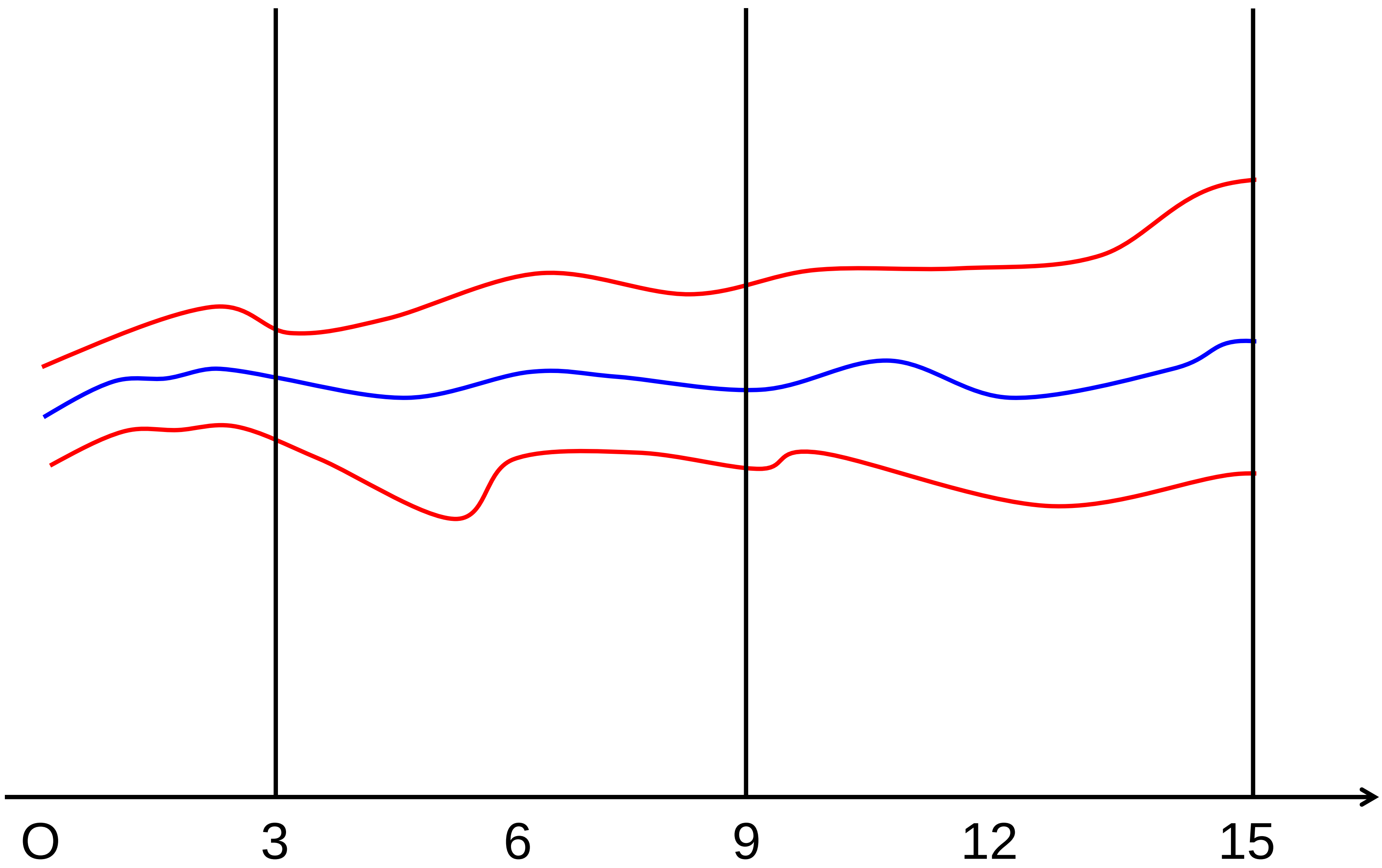
Ensemble Mean 4DVAR



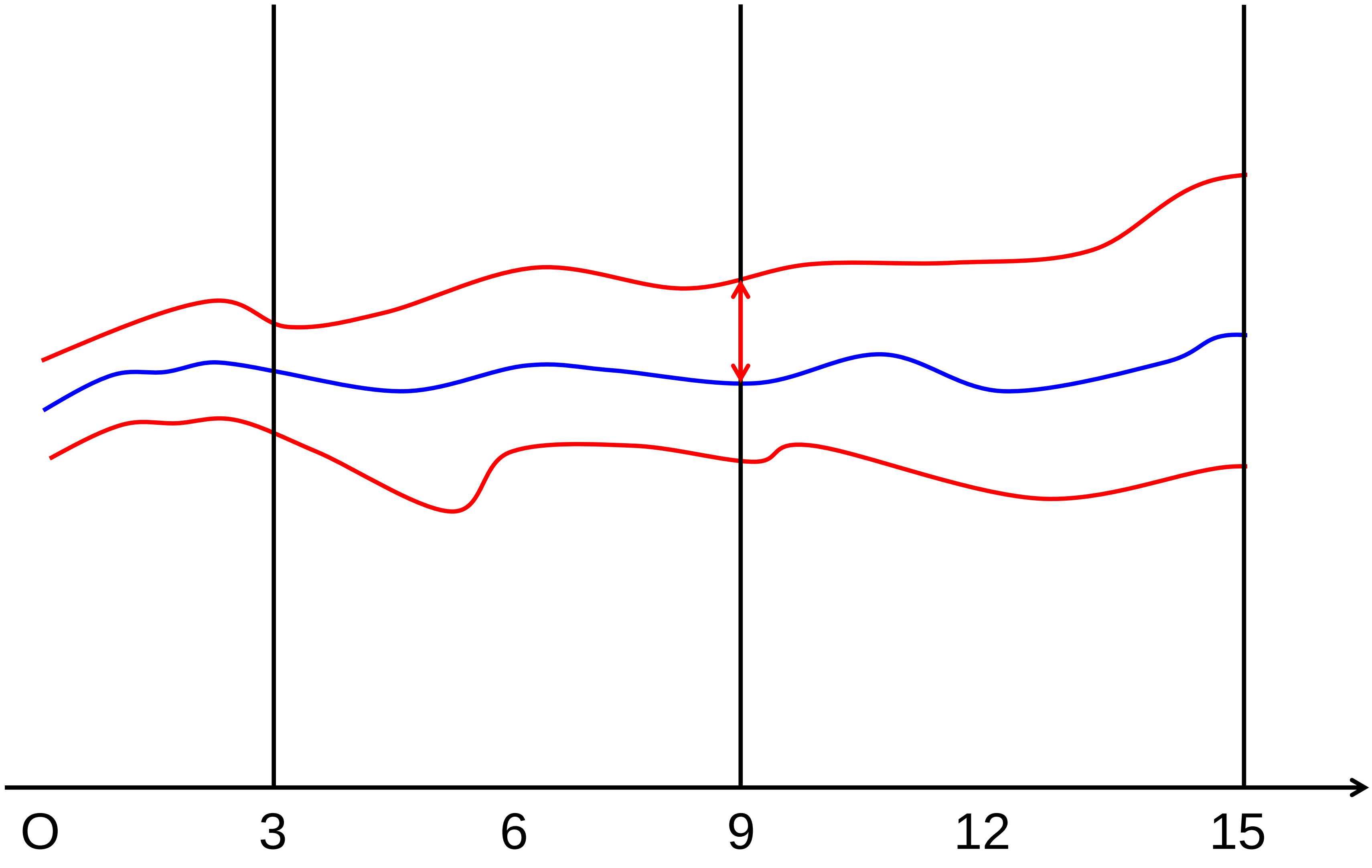
Ensemble Mean 4DVAR



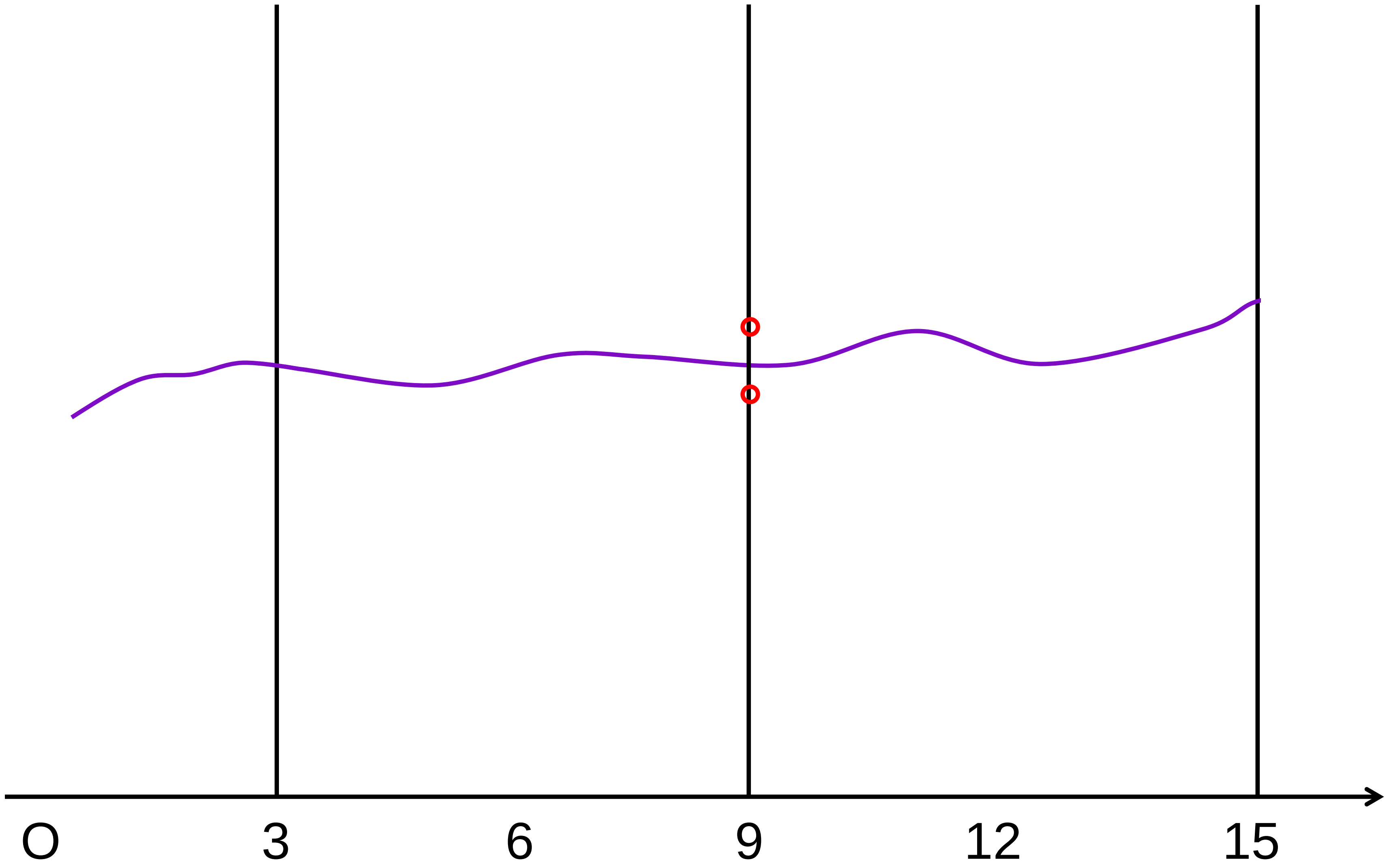
Ensemble Members



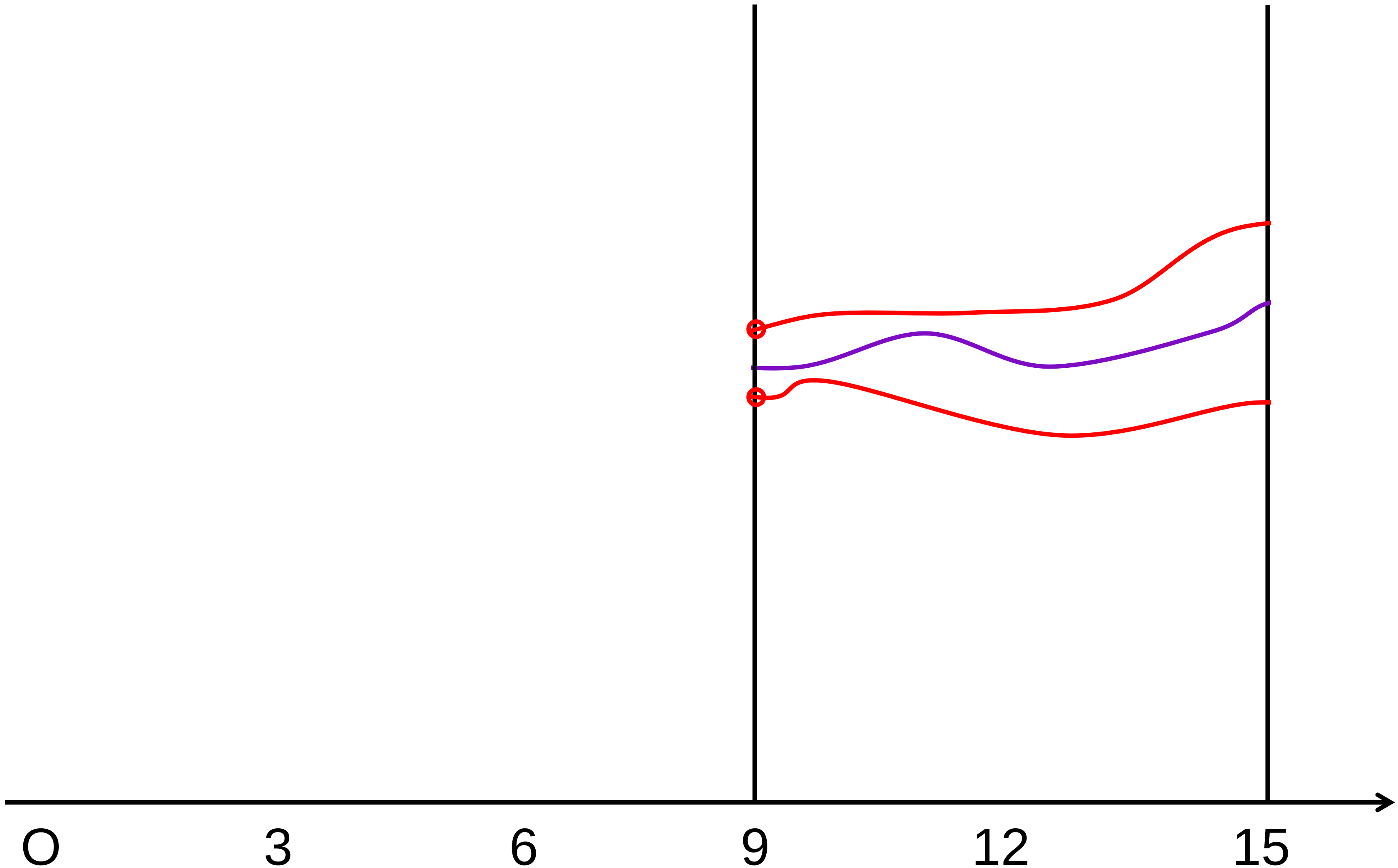
Ensemble Members



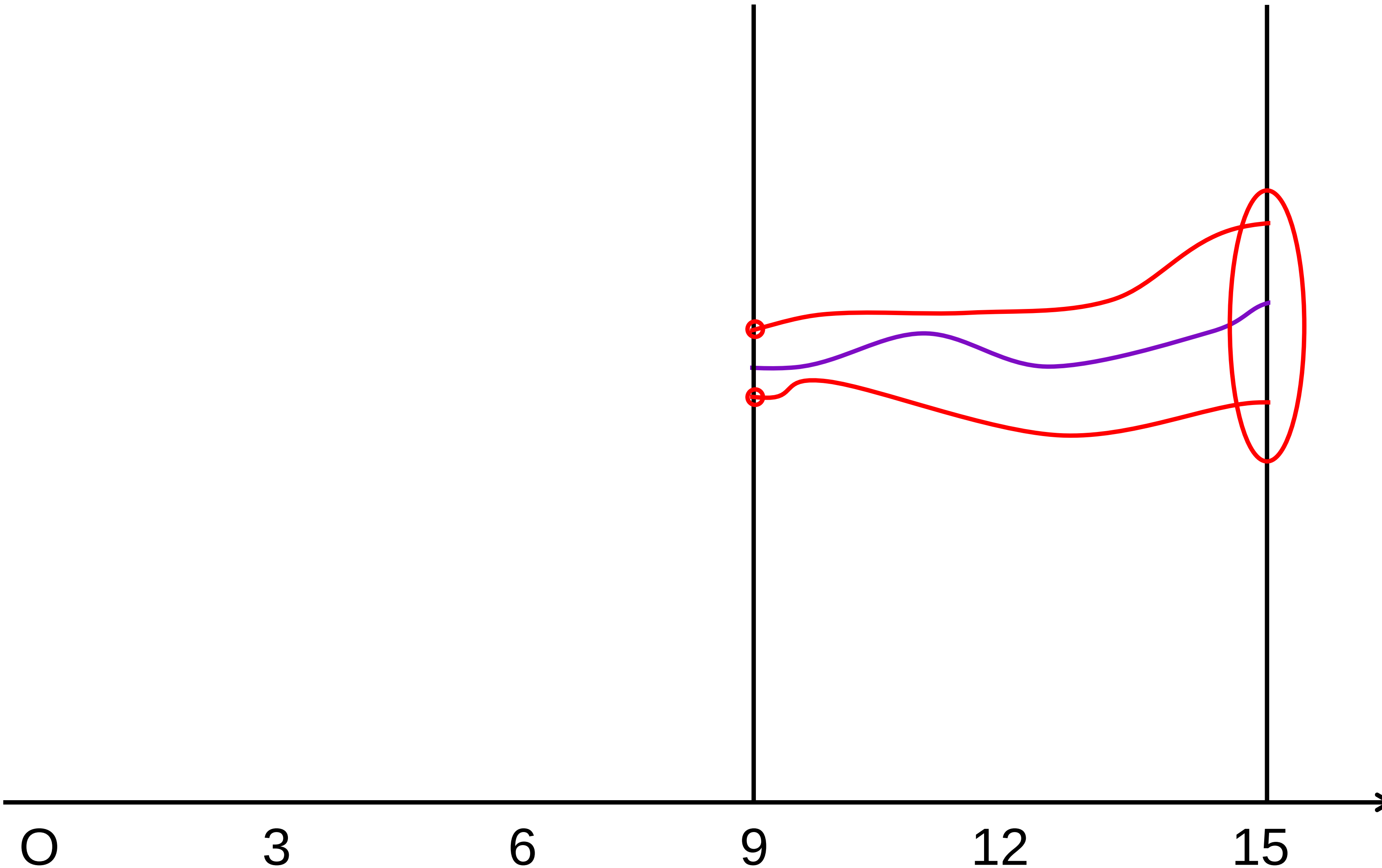
Ensemble Members



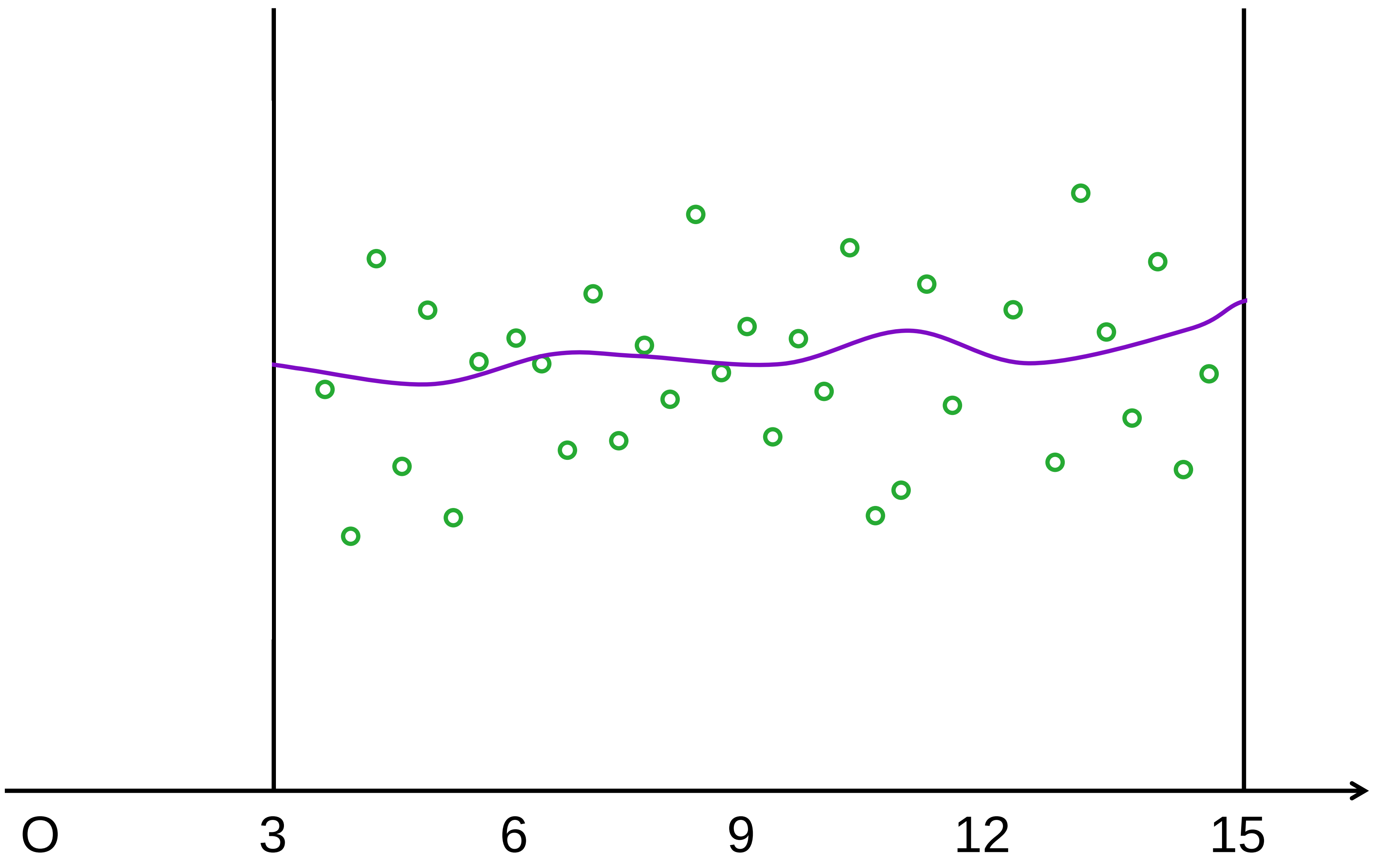
Ensemble Members



Spread



Skill



Quantity of available data

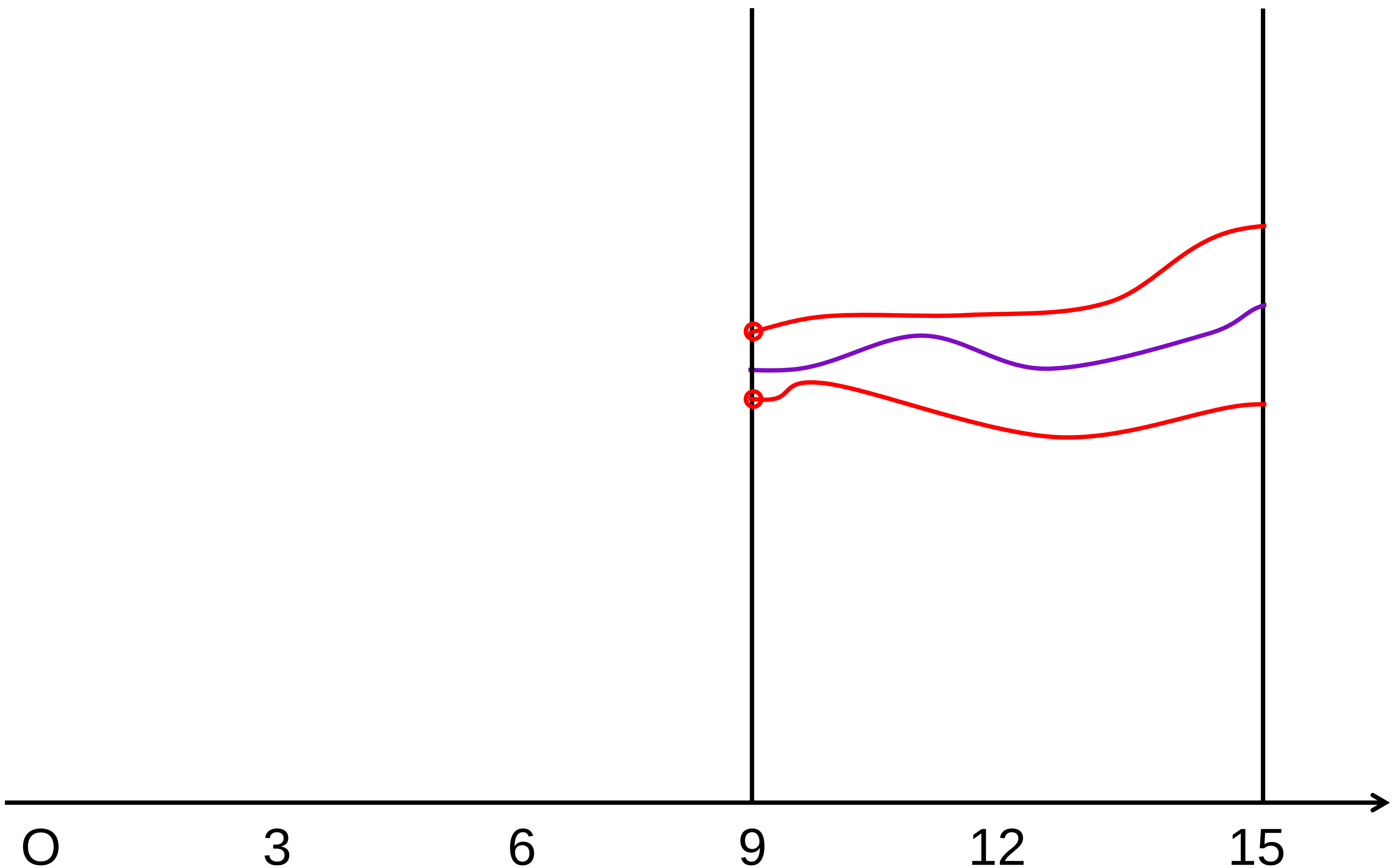
$$\text{Number of Observations} = M \sim 10^{20}$$

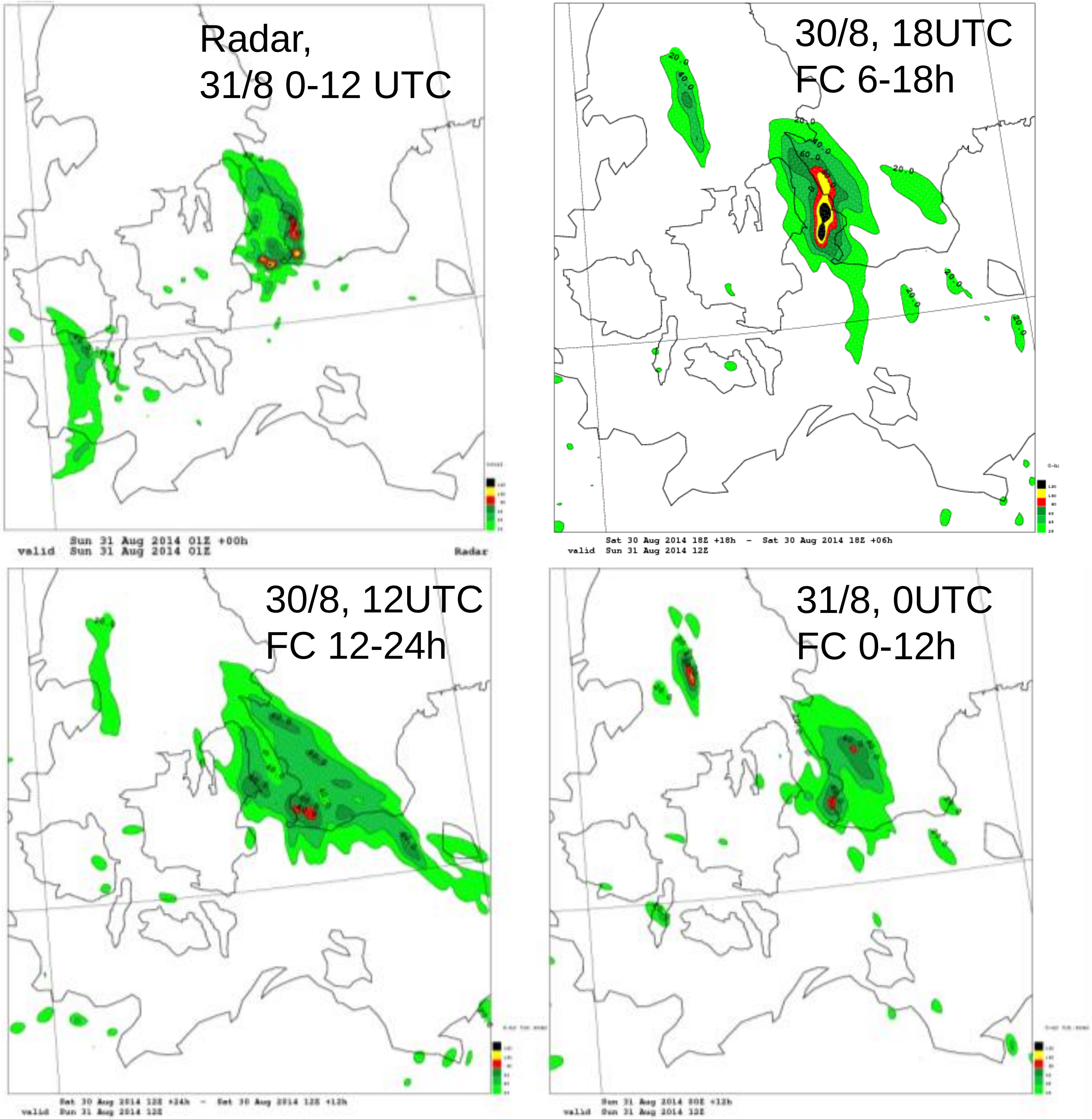
$$\text{Dimension of State Vector} = N \sim 10^{15}$$

M >> N

However

Only the largest scales are predictable at the end of the window


O
3
6
9
12
15


Radar,
31/8 0-12 UTC
Sun 31 Aug 2014 01Z +00h
valid Sun 31 Aug 2014 01Z
Radar
30/8, 18UTC
FC 6-18h
Sat 30 Aug 2014 18Z +18h - Sat 30 Aug 2014 18Z +06h
valid Sun 31 Aug 2014 12Z
30/8, 12UTC
FC 12-24h
Sat 30 Aug 2014 12Z +24h - Sat 30 Aug 2014 12Z +12h
valid Sun 31 Aug 2014 12Z
31/8, 0UTC
FC 0-12h
Sun 31 Aug 2014 00Z +12h
valid Sun 31 Aug 2014 12Z

Slutsatser

- **Synoptiska skalans stora betydelse**
- **Prediktabiliteten mycket begränsad hos mesoskalan**
- **Observationsunderlaget långt ifrån tillräckligt att beskriva mesoskalan**
- **Data assimilation och ensembler växer ihop → Många IC**